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THE UNIVERSITY OF ALBERTA

ADAPTIVE INFERENTIAL CONTROL FOR PROCESSES WITH MULTIRATE SAMPLING

BY

NATARAJAN M IYER

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

PROCESS CONTROL

THE DEPARTMENT OF CHEMICAL ENGINEERING

EDMONTON, ALBERTA

FALL, 1992

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Adaptive inferential control for processes with multirate sampling** submitted by **Natarajan M. Iyer** in partial fulfillment of the requirements for the degree of **Master of Science in Process Control**

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Dr. R. E Rink

DATE June 29, 1992

Dedicated to my mother

without whom this would not have been possible

ABSTRACT

This thesis examines the problem of controlling processes with intermittent measurements. Existing inferential control strategies are either suboptimal or based on restrictive assumptions. For example, some model based inferential control strategies utilize only approximate disturbance models to infer intersample values of the controlled output from secondary measurements. Other schemes estimate the unmeasurable controlled or 'primary' output assuming that the dynamics of the primary output are completely observable from secondary measurements.

This work makes three contributions. Firstly, it introduces a novel multirate structure for the Kalman filter, based on an optimal multirate Kalman filter design, used previously for multirate sample rate flight control. The multirate Kalman filter is used to infer intersample values of the primary output from secondary measurements. Secondly using innovations model analysis, the state space multirate Kalman filter formulation is transformed into a multirate input-output relationship, upon which an equivalent multirate minimum variance estimator is formulated. The estimator can be easily implemented in an adaptive framework as the multirate nature of the estimator model facilitates identification of the model parameters using available infrequent measurements of the primary output and frequent measurements of the secondary output. Finally, the multirate minimum variance

estimator is integrated with an implicit self-tuning multirate generalized minimum variance controller. The formulation of the minimum variance controller is such that identification of the multirate estimator model updates the controller parameters as well.

The adaptive multirate inferential estimation and control strategies are evaluated by MATLAB simulation on linear and nonlinear simulated models of a continuous stirred tank reactor. Simulation results show that these strategies perform significantly better in an adaptive framework compared to a fixed parameter framework, especially in the presence of unmeasured time varying disturbances and process nonlinearities. Recursive least squares is used to identify the the model parameters in the adaptive framework. A comparison of different modifications applied to the basic recursive least squares algorithm show that the covariance resetting scheme works best under the given circumstances. Suitable disturbance compensation techniques are also applied to the estimation algorithm to improve estimation accuracy and prevent the biasing of parameters in the presence of time varying load disturbances.

ACKNOWLEDGEMENT

I would like to thank Dr. R. K. Wood for his guidance and supervision during the course of this research. Thanks also due to Dr. S. L. Shah for patiently hearing out my ideas and providing me with valuable suggestions.

The work on this thesis was made more enjoyable by other graduate students. In particular, I owe much to discussions with Sreekanth Lalgudi, Christine Ho, Ravindra Gudi, Viral Maniar, Shaohua Niu, Munawar Saudagar and Pranob Banerjee.

I would also like thank the control group in general for providing a forum in which ideas could be exchanged in an intellectual yet friendly atmosphere. To Dr. Rink and Dr. Shah goes the credit of having really taught me my basics in Control.

The financial support from the Department of Chemical Engineering and the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

Finally, I take the opportunity to thank my parents, brother and sister in India for their constant support and encouragement over so many years.

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NOMENCLATURE

Alphabetical

$A(q^{-1})$	Polynomial corresponding to the primary output
$A_J(q^{-J})$	Multirate polynomial corresponding to the primary output in the primary multirate model
a_{Ji}	Coefficients of the polynomial $A(q^{-J})$
$\bar{A}(q^{-1})$	Polynomial corresponding to the secondary output in the secondary model
$\underline{\underline{a}}$	Matrix of steady state gains relating secondary outputs to unmeasured inputs (c.f. Eq. 2.1)
$\underline{\underline{a}}(s)$	Matrix of process transfer functions corresponding to the secondary process related to the unmeasured inputs
$a(s)$	Secondary process transfer function related to unmeasurable input in continuous time
$a(z^{-1})$	Secondary process transfer function related to unmeasurable input in discrete time
$\underline{\underline{A}}$	System model matrix
$\underline{\underline{A}}_{ij}$	Submatrix of $\underline{\underline{A}}$
a_i	Elements of $\underline{\underline{A}}$ corresponding to the primary subsystem states
$\bar{a}_i$	Elements of $\underline{\underline{A}}$ corresponding to the secondary subsystem states
$B(q^{-1})$	Polynomial corresponding to to input
$B_J(q^{-1})$	Polynomial corresponding to the input in the primary multirate model
b_{Ji}	Coefficients of the polynomial $B_J(q^{-1})$

$\bar{B}(q^{-1})$	Polynomial corresponding to the input in the secondary model
$\underline{b}$	Vector of steady state gains relating the primary output to the unmeasurable inputs
$\underline{b}(s)$	Vector of process transfer functions corresponding to the primary process related to the unmeasured inputs
$b(s)$	Primary process transfer function related to the unmeasured input in continuous time
$b(z^{-1})$	Primary process transfer function related to the unmeasured input in discrete time
$\underline{B}$	System model vector corresponding to the input vector
$\underline{B}_i$	Vector elements of $\underline{B}$
b_i	Elements of $\underline{B}$
$C(q^{-1})$	Polynomial corresponding to the noise sequence
$C_J(q^{-J})$	Polynomial corresponding the noise sequence in the primary multirate model
$\bar{C}(q^{-1})$	Polynomial corresponding to the noise sequence in the secondary model
c	Steady state gain relating primary output with the unmeasured input
$c(s)$	Primary process transfer function related to the measured input in continuous time
$c(z^{-1})$	Primary process transfer function related to the measured input in discrete time
$c^*(s)$	Primary process transfer function related to the measured input without time delay in continuous time
$c^*(z^{-1})$	Primary process transfer function related to the measured input without time delay in discrete time

C	Upper limit of trace of the covariance update matrix in the Variable forgetting factor algorithm
$\underline{C}_1$	System output vector for the primary subsystem
$\underline{C}_2$	System output vector for the secondary subsystem
$\underline{C}_1^+$	Pseudo-inverse of $\underline{C}_1$
$\underline{C}_2^+$	Pseudo-inverse of $\underline{C}_2$
C_a	Concentration of A in reactor
C_{af}	Concentration of A in feed to reactor
C_p	Specific heat of reaction mixture
d	Disturbance input
$D(s)$	Plant-model mismatch transfer function
$E\{.\}$	Expected value of statistic enclosed in parenthesis
e	Estimation or prediction error
e_y	Error in estimating y
e_v	Error in estimating v
$E_J(q^{-J})$	Multirate polynomial resulting from Diophantine identity in the minimum variance estimator for primary output
$\bar{E}(q^{-1})$	Polynomial similar to $E_J(q^{-J})$ in the minimum variance estimator for secondary output
E	Activation energy
$F(s)$	Filter transfer function in continuous time
$F(z^{-1})$	Filter transfer function in discrete time
$F_J(q^{-J})$	Polynomial resulting from Diophantine identity in the minimum variance estimator for the primary output
f_{Ji}	Coefficients of $F_J(q^{-J})$
$\bar{F}(q^{-1})$	Polynomial similar to $F_J(q^{-J})$ in the minimum variance estimator for the secondary output
$\bar{f}_i$	Coefficients of $\bar{F}(q^{-1})$

$\underline{\underline{F}}$	Continuous state model matrix
F_1	External force on spring (c.f. Eq. 3.1)
F	Feed flow rate to reactor
$\underline{f}$	Vector representing free response of reactor
f_i	Elements of $\underline{f}$
$G_I(s)$	Inferential controller transfer function in continuous time
$G_I(z^{-1})$	Inferential controller transfer function in discrete time
$G_c(s)$	Controller transfer function
$G_p(s)$	Process transfer function
$\hat{G}_p(s)$	Process model transfer function
$G_p^*(s)$	Process model transfer function without time delay
$G_L(s)$	Load disturbance transfer function
$G_{OL}(s)$	Open loop transfer function (c.f. Eq. 2.28)
$G_J(q^{-1})$	Polynomial product of $E_J(q^{-J})$ and $B_J(q^{-1})$
g_{Ji}	Coefficients of $G_J(q^{-1})$
$\bar{G}(q^{-1})$	Polynomial product of $\bar{E}(q^{-1})$ and $\bar{B}(q^{-1})$
$\bar{g}_i$	Coefficients of $\bar{G}(q^{-1})$
$\underline{\underline{G}}$	Continuous state model matrix corresponding to input vector
$\underline{g}$	Vector representing forced response of reactor
g_i	Elements of $\underline{g}$
$H_J(q^{-J})$	Polynomial corresponding to setpoint signal
ΔH	Heat of reaction
$\underline{\underline{h}}$	State output vector
$\underline{H}$	Measurement sensitivity vector for system
$\underline{H}_1$	Measurement sensitivity vector at the infrequent sampling instant
$\underline{H}_2$	Measurement sensitivity vector at the frequent sampling instants

h	Convective heat transfer cefficient
H	Dimensionless variable
$\underline{\underline{I}}$	Identity matrix
i	Integer
I_1	Control objective function employed when the control input is penalized
J	Integer; a multiple of the sampling period
J_{MV}	Minimum variance control objective function
$\underline{\underline{J}}^f$	Jacobian matrix of the free response
$\underline{\underline{K}}$	Kalman gain update matrix
$\underline{K}_{ij}$	Vector elements of $\underline{\underline{K}}$
k_i	Elements of $\underline{\underline{K}}$
k_1	Spring constant
$K_2(q^{-1})$	Polynomial corresponding to Kalman gain terms in $\underline{K}_{22}$ used in secondary innovations model
k	Discrete time delay
K_p	Proportional control constant
K_i	Integral time constant
k_o	Arrhenius rate constant
K_{min}	Minimum value for trace of $\underline{\underline{P}}$ in covariance resetting algorithm
$\underline{\underline{L}}_p$	Feedback gain matrix for predictive estimator
$\underline{\underline{L}}_c$	Feedback gain matrix for current estimator
$\underline{\underline{L}}$	Feedback gain matrix in the simple Kalman filter formulation (c.f. Eq. 3.19)
$L_J(q^{-1})$	Polynomial product of $E_J(q^{-J})$ and $M_J(q^{-1})$
l_{Ji}	Coefficients of $L_J(q^{-1})$
$\bar{L}(q^{-1})$	Polynomial product of $\bar{E}(q^{-1})$ and $\bar{M}(q^{-1})$

$\bar{l}_i$	Coefficients of $\bar{L}(q^{-1})$
$\underline{\underline{M}}$	Matrix whose elements are propagated values of covariance update matrix in Kalman filter formulation
$M(q^{-1})$	Polynomial corresponding to secondary output in primary model
$M_J(q^{-J})$	Polynomial corresponding to secondary output in multirate primary minimum variance estimator
m_{Ji}	Coefficients of $M_J(q^{-J})$
$\bar{M}(q^{-1})$	Polynomial corresponding to primary output in secondary minimum variance estimator
$\bar{m}_i$	Coefficients of $\bar{M}(q^{-1})$
N	number of sample intervals
n1	Total number of states in plant model given by Eq. 4.1
ny	Number of states in primary subsystem in plant model (c.f. Eq. 4.1)
n	Number of states in secondary subsystem in plant model (c.f. Eq. 4.3)
nv	Number of states corresponding to secondary process in Lu 's (1989) plant model (c.f. Eq. 3.27)
$\underline{p}$	Vector of steady state gains (c.f. Eq. 2.14)
$\underline{p}(s)$	Vector of secondary process transfer functions related to the measured inputs
p(s)	Secondary process transfer function related to unmeasured input in continuous time
$p(z^{-1})$	Secondary process transfer function related to unmeasured input in discrete time
$\underline{\underline{P}}$	Covariance update matrix
tr ($\underline{\underline{P}}$)	Trace of matrix $\underline{\underline{P}}$
$P(q^{-1})$	Transformation polynomial

$P_1(q^{-J})$	Auxilliary output transfer function
$P_N(q^{-J})$	Numerator polynomial of $P_1(q^{-J})$
$P_D(q^{-J})$	Denominator polynomial of $P_1(q^{-J})$
p_n	Coefficient of $P_N(q^{-J})$
p_d	Coefficient of $P_D(q^{-J})$
Q	Penalty weight associated with control action in the objective function I_1
$\underline{\underline{Q}}$	Pre-specified matrix to which the covariance update matrix is reset in the covariance resetting algorithm
q	Trace of $\underline{\underline{Q}}$
$\underline{r}$	Vector of residuals (c.f. Eq. 2.9)
$\underline{\underline{R}}_{\xi}$	Noise covariance matrix corresponding to noise vector $\underline{\xi}$
$\underline{\underline{R}}_{v}$	Noise covariance matrix corresponding to output measurement noise vector
$\underline{R}$	System model vector corresponding to white noise sequence w
$\underline{\underline{R}}_{w}$	Covariance matrix for to white noise sequence w
$\underline{\underline{R}}_{z}$	Covariance matrix for measurement noise vector $\underline{\omega}$
R_y	Variance of primary measurement noise
R_v	Variance of secondary measurement noise
$\underline{R}_i$	Vector elements of $\underline{R}$
r_i	Elements of $\underline{R}$
R	Universal gas constant
s	Laplace or continuous time domain variable
T	Basic sampling period (c.f. Section 3.7.1)
T_R	Temperature of reactor
T_c	Temperature of coolant
T_f	Temperature of feed to the reactor
T_{ai}	Time constants in the secondary process transfer function

T_{bi}	Time constants in the primary process transfer function
T(s)	Transfer function in Eq. 2.24
T'(s)	Transfer function similar to T(s) (c.f. Eq. 2.41)
t	Time
t_0	Initial time
u	Measured input or control input
u_c	Filtered input in Eq. 2.73
u^1	Model following control input
$\underline{u}$	Measured input vector
$\underline{\underline{U}}$	Matrix whose elements are derivatives of $\underline{u}$
v	Secondary output
$\underline{v}$	Output vector
$\underline{v}_0$	Initial output vector
$\underline{\underline{V}}$	Matrix whose elements are derivatives of vector $\underline{v}$
V	Volume of reactor
$\underline{\underline{W}}$	Upper bound for the covariance update matrix in the variable forgetting scheme
w(t)	White noise sequence
x_i	State or variable
$\underline{x}$	State vector
$\underline{x}^1$	State vector for primary subsystem
$\underline{x}^2$	State vector for secondary subsystem
y	Primary output
y_{sp}	Set point for primary output
z	Variable in discrete domain

Greek

$\underline{\alpha}$	Vector of inferential gains
$\underline{\alpha}(s)$	Vector of dynamic inferential estimator transfer functions
$\underline{\alpha}(z^{-1})$	Vector of dynamic inferential estimator transfer functions
$\alpha(s)$	Dynamic inferential estimator transfer function
$\alpha(z^{-1})$	Dynamic inferential estimator transfer function
β	Dimensionless variable in continuous stirred tank reactor model
β^1	Forgetting factor related to disturbance compensation algorithm
δ	Time delay mismatch
ε	Prediction error during identification
$\underline{\underline{\Phi}}$	Discrete time state model matrix
$\underline{\underline{\Phi}}'$	Discrete time state model matrix similar to $\underline{\underline{\Phi}}$
$\underline{\Phi}$	Regression vector
$\underline{\Phi}_y$	Regression vector corresponding to the primary multirate estimator
$\underline{\phi}_v$	Regression vector corresponding to the secondary estimator
γ	Dimensionless variable in continuous stirred tank reactor model
$\underline{\underline{\Gamma}}$	Discrete time state input coefficient vector
$\underline{\underline{\Gamma}}'$	Discrete time state input coefficient vector similar to $\underline{\underline{\Gamma}}$
η	Disturbance
$\underline{\eta}$	Disturbance vector
Ψ	Auxilliary output corresponding to the primary output
λ_i	i^{th} eigenvalue of the polynomial $A(q^{-1})$
$\lambda(t)$	Variable forgetting factor

μ	Function defined in Eq 5.38
$\underline{\nu}$	Vector of unmeasured disturbances
ν_i	Elements of $\underline{\nu}$
ν_v	Estimation error associated with the secondary output
ν_y	Estimation error associated with the primary output
$\underline{\theta}$	Parameter vector
$\underline{\theta}_y$	Parameter vector related to the primary multirate estimator
$\underline{\theta}_v$	Parameter vector related to the secondary estimator
$\underline{\underline{\Theta}}$	Matrix defined in Eq. 3.10
ρ	Density of reaction mixture
σ	Noise covariance matrix in the Variable forgetting factor algorithm
σ_u	Filter coefficient in Eq. 2.73
τ	Dimensionless variable in continuous stirred tank reactor model
τ_d	Continuous time delay
τ_f	Continuous time filter time constant
ω	Frequency
$\underline{\omega}$	Vector of measurement noise
ω_y	Measurement noise associated with primary measurement
ω_v	Measurement noise associated with secondary measurement
$\underline{\xi}$	Vector of process noise
ξ	Process noise
ξ_y	Prediction error associated with primary output
ξ_v	Prediction error associated with secondary output

Subscripts

$\underline{\underline{\phi}}_{ij}$	Covariance matrix associated with vectors $\underline{i}$ and $\underline{j}$
$\tilde{y}$	Change in y only due to unmeasured inputs (c.f. 2.15)
ss	Steady state value

Superscripts

$\underline{\underline{x}}^T$	Transpose of $\underline{\underline{x}}$
$\tilde{x}$	Deviation of x from its steady state value
$\bar{x}$	Prediction estimate of state x (c.f. Eq. 3.14)
$\hat{x}$	Estimate of state x; also the current estimate of x as defined by Eq. 3.16
$\underline{x}^f$	Vector $\underline{x}$ after filtering
y^1	Value obtained after sutracting the effects of disturbances from y
$\underline{v}^{(i)}$	i^{th} derivative of vector $\underline{v}$
y_{sp}^{sc}	Scaled set point

Abbreviations

DMC	Dynamic matrix control
IMC	Internal model control
MAC	Model algorithmic control
MPM	Model-plant mismatch
MV	Minimum variance control
PI	Proportional plus integral control
RLS	Recursive least squares

CHAPTER 1 INTRODUCTION

Most control system designs rely on measurements of the controlled output to correct for variations of the output from the specified set point. Quite often, these measurements are not available at the desired sampling rate. Under these circumstances, one alternative is to use indirect measures of the controlled output that can be measured at the desired rate. For example, in distillation column control, selected liquid tray temperature measurements are frequently used to control product compositions in the absence of on-line composition analyzers or possibly in conjunction with analyzers if the time delay associated with the analysis is very long. Controlling an indirect variable, however does not guarantee the desired control of the variable of interest. Alternatively, one may have to control a number of indirect measures to achieve the desired results. Theoretically, under such circumstances, it would be easier to infer the desired output from all such indirect measures and control the process using the inferred estimate. The class of process control strategies that use an inferred estimate of the desired output to control the process are referred to as inferential control strategies. Inferential control was defined as such, first by Brosilow (1978) to describe his model based approach that used estimates of the controlled output obtained by inferring the effects of unmeasured disturbances on the controlled or 'primary' output using indirect

or 'secondary' measurements to control the process. The philosophy and shortcomings of the model based strategies that followed soon after Brosilow's work in 1978 are discussed in detail in Chapter 2.

State estimator designs that estimate unknown states of the process from available measurements may also be used in an inferential control framework. A review of existing state estimator designs used for inferential estimation is presented in Chapter 3.

The Kalman filter is one such state estimator design which estimates the unknown states optimally from all available measurements in the presence of process and measurement noise. Glasson (1980) in a report prepared for the Analytic Science Corporation outlines a multirate Kalman filter design which optimally infers the infrequently measured output from secondary measurements in the context of multirate sampled flight control system design. This multirate Kalman filter design is adapted with some modifications for the purpose of inferential estimation. Innovation model analysis is used to transform the suboptimal multirate Kalman filter into a suitable multirate autoregressive model.

A minimum variance estimator based on the multirate regressive model is used for inferential estimation. The multirate structure of the inferential estimator facilitates the identification of the model parameters (in an adaptive framework) using the frequent measurements of the secondary output and the infrequent measurements of the primary output. A multirate generalized minimum variance control scheme is developed to be implemented in an

adaptive framework with the multirate inferential estimator. The implementation aspects of the adaptive multirate inferential controller are considered in Chapter 5 when the estimation and control algorithms developed in Chapter 4 are applied to linear and nonlinear models of a continuous stirred tank reactor.

The theoretical implications of the multirate Kalman filter and its potential application to chemical and biochemical processes as a software sensor are discussed in the Chapter 6.

CHAPTER 2 SUBOPTIMAL MODEL BASED INFERENTIAL CONTROL METHODS

2.1 Introduction

The inferential control strategy was originally developed by Brosilow and co-workers(1978 a,b,c) to deal with control problems where measurements of the controlled output were not available. For example, in distillation column control, often the product compositions which are the controlled outputs of the process cannot be measured. In such cases, the proposed strategy allows the unmeasurable controlled output to be inferred from secondary measurements such as tray temperatures .

The following is an outline of the philosophy of the proposed strategy. Measurements of the controlled output are usually required in any control system to counteract the effects of unmeasured disturbances , since effects of measurable disturbances can always be compensated by feedforward control. In the absence of measurements of the controlled or primary output, an inferential control system provides control for the unmeasured disturbances by inferring their effects on the primary output from secondary measurements.

In this chapter, we focus on the development of steady state and dynamic estimators based on the theory outlined by Brosilow and coworkers. The robustness aspect of inferential control mechanisms is analyzed by comparing their performance in the

presence of model-plant mismatch with the performance of the Smith predictor. We finally consider an adaptive inferential control strategy based on the above philosophy, developed for processes with intermittent measurements.

2.2 Steady state estimator design

Most chemical process control problems share the following characteristics: the process is subject to unmeasurable disturbances that vary slowly; the primary output to be controlled is not easily measured, and there are other process outputs that are relatively easily measured.

The input disturbances frequently arise because of changes in the operation of units upstream from the process. For small input disturbances, the process can be assumed to behave linearly.

Let the unmeasured output y and the measurement vector $\underline{v}$ be related to the input disturbance vector $\underline{\nu}$ as follows

$$\underline{v} = \underline{\underline{a}}^{T} . \underline{\nu} \qquad 2.1$$

$$y = \underline{b}^{T} . \underline{\nu} \qquad 2.2$$

When the number of unmeasured inputs is equal or less than the number of measurements , one can solve exactly for $\underline{\nu}$ from equation 2.1 and compute y exactly from equation 2.2. In this case

$$y = \underline{b}^{T} \underline{\underline{a}}^{-1} . [\ \underline{v}\ ] \qquad 2.3$$

This approach was taken by Weber and Brosilow (1972).

2.2.1 Least squares based static estimator

The least squares approach proposed by Joseph and Brosilow (1978 a) looks at a more realistic situation where the number of unmeasured inputs is unlimited.

If $\underline{\nu}$ can be modelled as a zero mean random vector, then $\underline{v}$ and y are also random and the problem reduces to the estimation of one random variable y in terms of another $\underline{v}$.

The least squares calculation minimizes the expectation of the squared error i.e $E\{(y - \hat{y})^T(y - \hat{y}) \}$. So, if the input disturbances are Gaussian, then the least squares estimate of y would be given by (Joseph and Brosilow, 1978 a)

$$\hat{y} = (\underline{\phi}_{yv} \underline{\phi}_{vv}^{-1}) \cdot \underline{v} \qquad 2.4$$

where $\underline{\underline{\phi}}_{vv} = E \{ \underline{v}^* \underline{v}^T \} = \underline{\underline{a}}^T * \underline{\underline{a}}$

and $\underline{\phi}_{yv} = E \{ y^* \underline{v}^T \} = \underline{b}^T * \underline{\underline{a}}$

From equation 2.4, the least squares estimate $\hat{y}$ would be given by

$$\hat{y} = \underline{\alpha}^T . \ \underline{v} \qquad 2.5$$

where $\underline{\alpha} = [\underline{\underline{a}}^T \underline{\underline{a}}]^{-1} \ \underline{\underline{a}}^T \underline{b}$ 2.6

The constant vector $\underline{\alpha}$ can also be obtained from the least squares solution of

$$\underline{\underline{a}}\ \underline{\alpha} = \underline{b} \qquad 2.7$$

From equations 2.1 and 2.5, we can write

$$\hat{y} = \underline{\alpha}^T . \underline{\underline{a}}^T . \underline{\nu} \qquad 2.8$$

So, from equations 2.2 and 2.8, the estimation error would be given by

$$\begin{aligned} e &= y - \hat{y} \\ &= [\ \underline{b}^T - \underline{\alpha}^T \underline{\underline{a}}^T\ ]\ \underline{\nu} = \underline{r}^T \underline{\nu} \end{aligned} \qquad 2.9$$

The expected values of the error and error squared are

$$\begin{aligned} E\{ e \} &= 0 \\ E\{ e^T e \} &= \underline{r}^T \underline{r}\ \underline{\Phi}_{\nu\nu} \end{aligned} \qquad 2.11$$

So, without loss of generality, we can assume $\underline{\Phi}_{\nu\nu} = \underline{\underline{I}}$, in which case equation 2.11 states that the expected mean-square error is zero if and only if $\underline{b} = \underline{\underline{a}}\ \underline{\alpha}$.

2.2.2 Effect of measured variables

Including changes in the measured variable u that affect the process, the model described by equations 2.1 and 2.2 can be rewritten as

$$\underline{v} = \underline{\underline{a}}^T . \underline{\nu} + \underline{p} . u \qquad 2.12$$

$$y = \underline{b}^T . \underline{\nu} + c . u \qquad 2.13$$

Defining new variables

$$\underset{\sim}{v} = \underline{v} - \underline{p} . u \qquad 2.14$$

$$\underset{\sim}{y} = y - c . u \qquad 2.15$$

the equations derived earlier can be applied to the estimation of $\underset{\sim}{y}$ from $\underset{\sim}{v}$, and the solution is

$$\hat{y} = \underline{\alpha}^T . [\ \underline{v} - \underline{p} . u\ ] + c . u \qquad 2.16$$

2.2.3 Selection criteria for measurements

In the least squares approach , the number of measurements need not be greater than the number of unmeasured inputs. Hence, the criteria for selecting secondary measurements must be specified.

Two conditions for good estimator performance according to Joseph and Brosilow, 1978a are low relative error and low model condition number of the matrix: $[\underline{\underline{a}}^T a]$.

Measurements are selected such that the measurement set that yields the lowest relative estimation error, without the model condition number exceeding a certain prespecified value is chosen. Morari and Stephanopoulus (1980) suggested a procedure to choose measurements such that the following relative squared error criterion

$$\frac{E\{ (y - \hat{y})^T (y - \hat{y}) \}}{E\{ y^T y \}}$$

is minimized.

2.3 Dynamic inferential control system

Since it is generally necessary to add some form of dynamic compensation to the static estimation scheme discussed earlier, the static estimator given by equation 2.16 is incorporated into a dynamic control system which minimizes output feedback as shown in Figure 2.1.

The difference between the static estimator and the dynamic inferential estimator $\underline{\alpha}^T(s)$ is the inclusion of simple lead-lag elements in the static estimator design to compensate for the load dynamics of the primary and secondary processes. The structure of the inferential control system shown in Figure 2.1 is similar to that of a feedforward control system. As in a feedforward control system, the controller has two inputs: the set point and an estimate of the disturbance effects.

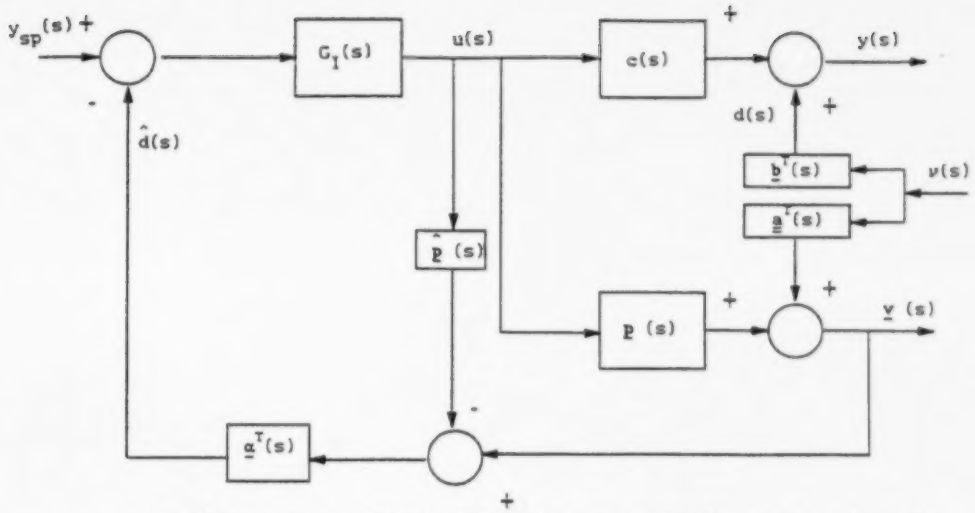

Figure 2.1 Block diagram representation of a dynamic inferential control system

The controller $G_I(s)$ manipulates the control effort to track the set point and to produce a cancellation effect $-\hat{d}(s)$ for the disturbances affecting the primary process. The cancellation can be expected to be perfect if the controller dynamics are an exact inverse of the proceess dynamics; i.e if $G_I(s) = c^{-1}(s)$ and if the inference of the disturbance effects is accurate; i.e if $\hat{d}(s) = d(s)$.

The following factors can be considered to be important to achieve desirable performance while designing the inferential control system (Brosilow and Tong, 1978 b):

1) Model-plant mismatch

Smaller values of $(\underline{p}(s)-\hat{\underline{p}}(s))$ would yield a better approximation of $\hat{d}(s)$ to $d(s)$. When $\hat{\underline{p}}(s) \neq \underline{p}(s)$, the parameters of $\hat{\underline{p}}(s)$ are adjusted to assure negative feedback of the disturbance effects which might prevent instability. In the presence of model uncertainity, the robustness of the inferential control system can be improved by designing a low pass filter that increases the margin of stability. This aspect of the inferential control system is discussed in detail later in this chapter.

2) Controller design

The system will be stable provided the original process is stable and the controller $G_I(s)$ is stable. Since the controller is designed to be an inverse of the process, this means that if one or more elements of the process is nonminimum phase, it will not be possible to implement an exact inverse of the process. Thus in general, $G_I(s)$ will be designed to be a stable approximation to the inverse of the process with its steady state gain chosen such that $G_I(0)\ c(0) = 1$.

3) Inferential estimator design

Suboptimal estimators used in the dynamic inferential control system described in Figure 2.1 are of the form

$$\underline{\alpha}(s) = \left[\alpha_{10} \frac{T_{a1}s + 1}{T_{b1}s + 1} , \quad \alpha_{20} \frac{T_{a2}s + 1}{T_{b2}s + 1} , \ldots\ldots\ldots \right]$$

The constants α_{10}, α_{20}.... are those asscciated with the optimal steady state estimator. The time constants T_{a1}, T_{b1}, .. associated with the lead and lag elements are determined such that the expected error in estimation is minimized.

Brosilow and co-workers have suggested ad-hoc methods to incorporate dynamic elements into the estimator structure. The following example is reproduced from Brosilow and Tong (1978 b) to illustrate the estimator design procedure.

Example 2.1

The example process was a sixteen stage, five component distillation column with a total condenser and a total reboiler that had been previously used by Weber and Brosilow (1972). In the study of Brosilow and Tong (1978 b) the column overhead butane composition y is estimated from a single tray temperature measurement v, for the case of three unmeasurable inputs identified as: ν_1, ν_2, ν_3. The linearized column model, obtained from step tests on a simulation of the full nonlinear model, was represented as

$$y(s) = \underline{b}^T(s)\, \underline{\nu}(s) + c(s)\, u(s) \qquad 2.17$$

$$v(s) = \underline{\underline{a}}^T(s)\, \underline{\nu}(s) + \underline{p}(s)\, u(s) \qquad 2.18$$

where $\underline{\nu}(s) = [\nu_1, \nu_2, \nu_3]^T$

$\underline{b}^T(s) = [b_1, b_2, b_3]$

$\underline{\underline{a}}^T(s) = [a_1, a_2, a_3]$

The steady state gains of the disturbance transfer functions $\underline{b}^T(s)$ and $\underline{\underline{a}}^T(s)$ are denoted by the vectors $\underline{b}^T$ and $\underline{\underline{a}}^T$ respectively. The various parameters of the model are given in Table 2.1.

Table 2.1 Transfer functions relating effect of the unmeasured inputs on overhead butane composition and temperature on plate 14

ν	ν_1	ν_2	ν_3
	$b_1(s)$	$b_2(s)$	$b_3(s)$
y	$\frac{-0.188}{72s + 1}$	$\frac{0.002}{85s + 1}$	$\frac{0.00043}{80s + 1}$
	$a_1(s)$	$a_2(s)$	$a_3(s)$
v	$\frac{-42.02}{50s + 1}$	$\frac{0.46}{75s + 1}$	$\frac{1.10}{70s + 1}$

The estimator gain or the steady state inferential gain, sincethere is only one temperature measurement, is calculated according to equation 2.6 as

$$\alpha = [\underline{\underline{a}}^T \underline{\underline{a}}]^{-1} \underline{\underline{a}}^T \underline{b} = 0.0045 \qquad 2.19$$

The dynamic elements of the estimator are selected such that the dynamic error in estimation e(s) is minimized.

$$\begin{aligned} e(s) &= [\underline{b}^T(s) - \alpha(s)\, \underline{\underline{a}}^T(s)].\underline{\nu}(s) \\ &= \underline{r}^T(s)\, \underline{\nu}(s) \end{aligned} \qquad 2.20$$

If $\underline{r}^T(s)$ is minimized so that it is exactly zero, then the estimate is perfect. However, for this particular system, since there are three unmeasured inputs, there is no one choice of the estimator $\alpha(s)$ which will yield perfect dynamic estimates.

Since the range of time constants for the primary and secondary process transfer functions is not large, a reasonable ad-hoc choice for the magnitude of the lead time constant of the estimator is the average of the time constants in $\underline{\underline{a}}(s)$. Similarly, the lag time constant in $\alpha(s)$ is the average of the time constants in $\underline{b}(s)$. This results in

$$\alpha(s) = \alpha \frac{(66s + 1)}{(77s + 1)} \qquad 2.21$$

and substituting α from equation 2.19 gives

$$\alpha(s) = 0.0045 \frac{(66s + 1)}{(77s + 1)} \qquad 2.22$$

2.4 Inferential control and the Smith predictor

The Smith predictor (Smith, 1957) was developed to provide time delay compensation to improve the performance of systems with large time delays. Since the Smith predictor is a model-based controller, it bears a strong resemblance to the structure of the inferential control system (Brosilow, 1979). Although the Smith predictor is useful to control load disturbances, the simulation study of Meyer et al (1976) showed that a conventional PI controller could provide better regulatory control than the Smith

predictor under certain conditions. Brosilow(1979) compared the regulatory characteristics of the inferential control system with those of the Smith predictor and concluded that it was always possible to design an inferential controller to perform as well as or better than a Smith predictor for any stable process.

2.4.1 Inferential control versus the Smith predictor

The generalized inferential control strategy shown in Figure 2.2, proposed by Brosilow (1979), bears a strong resemblance to the Smith predictor presented in Figure 2.3. The structural difference between the two is that the controller $G_I(s)$ in Figure 2.2 is equivalently implemented by the feedback loop around the controller $G_c(s)$ in Figure 2.3.

That is

$$G_I(s) = (1 + G_c(s)G_p^*(s))^{-1} \; G_c(s) \qquad 2.23$$

Essentially, the difference between the two systems lies in the fact that while the Smith predictor is a closed loop control system, where the controller $G_c(s)$ is typically a standard PID control algorithm, the inferential controller is an open loop controller where $G_I(s)$ is a stable inverse of the process transfer function.

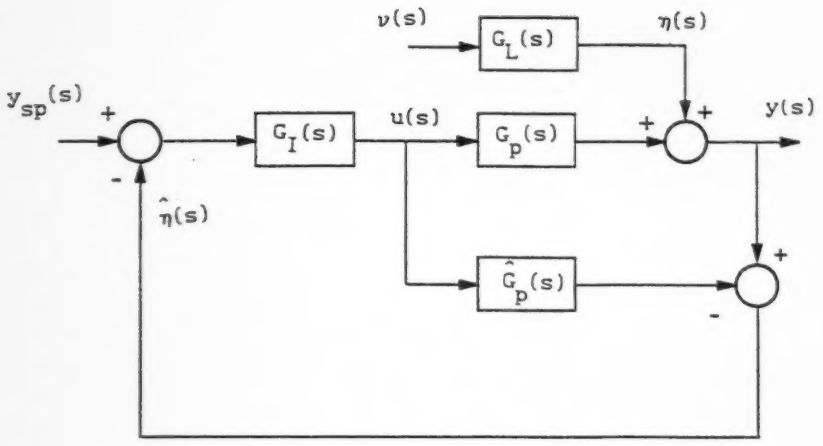

Figure 2.2 Block diagram representation of the generalized inferential control strategy of Brosilow (1979)

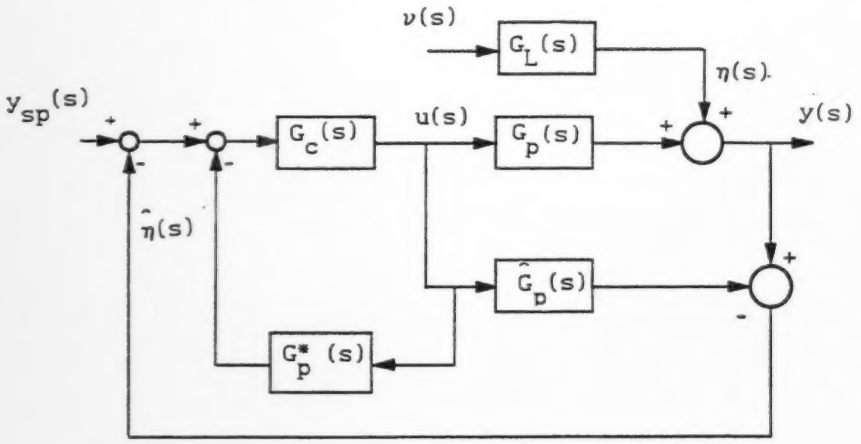

Figure 2.3 Block diagram representation of the Smith predictor control strategy

2.4.2 Algebraic properties of the inferential controller

The response of $y(s)$ to $\nu(s)$ and $y_{sp}(s)$ for the system given in Figure 2.2 is

$$y(s) = G_p(s)\, G_I(s)\, T(s)\, y_{sp}(s) + [\, 1 - G_p(s)\, G_I(s)\, T(s)]\, \eta(s) \qquad 2.24$$

where $$T(s) = [\, 1 + (G_p(s) - \hat{G}_p(s))\, G_I(s)\,]^{-1}$$

$$\hat{\eta}(s) = T(s)\, \eta(s) \qquad 2.25$$

$$\eta(s) = G_L(s)\, \nu(s) \qquad 2.26$$

From equation 2.24, it can be seen that the estimate of the unmeasured disturbances is distorted when $\hat{G}_p(s) \neq G_p(s)$. From equation 2.24, we also note that if $G_I(s) = \hat{G}_p^{-1}(s)$, we have $y(s) = y_{sp}(s)$ for any $\nu(s)$, i.e. perfect control. However, this is true only if a stable and realizable inverse of the process exists. Brosilow (1979) and Garcia and Morari (1985) proposed stable approximations of the process inverse and also suggested the use of additional disturbance rejection filters which assured the stability of the control system and insensitivity to modelling errors even with imperfect process models.

2.4.3 Design of disturbance filter to counter modelling errors and unmeasured disturbances

A commonly expressed concern about the performance of the Smith predictor (Seborg et al., 1989) is its degradation in the presence of modelling errors. In order to reduce the sensitivity of the Smith predictor to modelling errors, it is often necessary to detune the controller thereby degrading the performance of the control system.

Generally stated, the controller design of an inferential control system is divided into two stages. In the first stage, one designs the controller as a stable approximation of the inverse of the process transfer function, assuming that the process model is a perfect representation of the process. In the second stage, one designs a controller filter F(s) to account for expected modelling errors. The relationship between the estimated disturbance $\hat{\eta}$ to the actual disturbance η, as a result of incorporating the filter into the design of the inferential control system as shown in Figure 2.4 is given by

$$\hat{\eta}(s) = \frac{F(s)}{[1 + F(s)D(s)G_I(s)]} \cdot \eta(s) \qquad 2.27$$

$$D(s) = (G_p(s) - \hat{G}_p(s))$$

The characteristic equation for the transfer function given in equation 2.27 can be written as

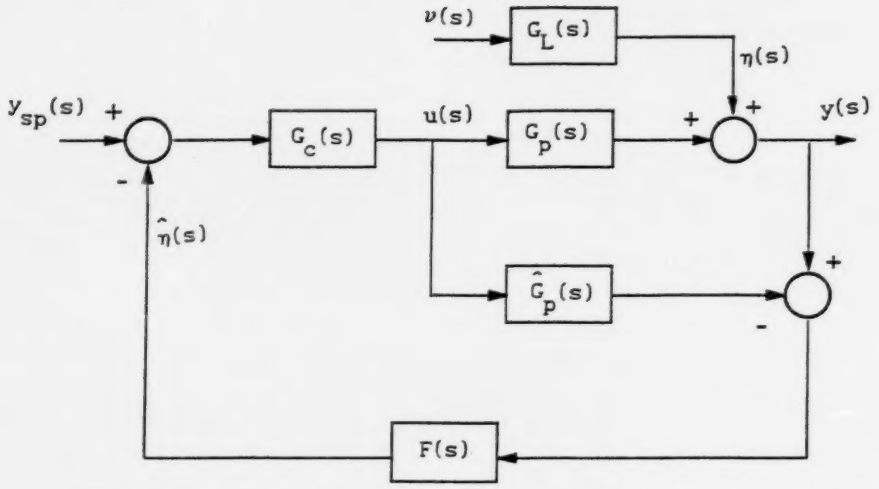

Figure 2.4 Block diagram representation of a generalized inferential control system which includes a disturbance filter

$$1 + G_{OL}(s) = 0 \qquad 2.28$$

where $G_{OL}(s)$ is the equivalent open loop transfer function given by

$$G_{OL}(s) = F(s)D(s)G_I(s) \qquad 2.29$$

2.4.4 Effect of time delay mismatch

For a Smith predictor, it has been reported (Schleck and Hanesian, 1978) that if the assumed time delay is not within 30 % of the actual process time delay, the predictor is inferior to a PI controller with no time delay compensation.

of the actual process time delay, the predictor is inferior to a PI controller with no time delay compensation.

For the inferential control system presented in Figure 2.4, if we consider a simple first order plus time delay process,

$$G_p(s) = \frac{K \quad e^{-\tau_d s}}{(\tau_p s + 1)} \qquad 2.30$$

and the process model is assumed to be

$$\hat{G}_p(s) = G_p^*(s)\, e^{-\hat{\tau}_d s}$$

where $$G_p^*(s) = \frac{K}{(\tau_p s + 1)} \quad , \quad \hat{\tau}_d \neq \tau_d \qquad 2.31$$

then the characteristic open loop transfer function relating the estimated disturbances $\hat{\eta}$ to the actual disturbances η , would be given by equation 2.29 as (assuming $G_I(s) = G_p^{*-1}(s)$)

$$G_{OL}(s) = F(s)\, [e^{-(\tau_d - \hat{\tau}_d)s} - 1\,]\, e^{-\hat{\tau}_d s} \qquad 2.32$$

and if F(s) is taken as unity, then the characteristic denominator transfer function will become

$$G_{OL}(s) = (e^{-\delta s} - 1).\ e^{-s\hat{\tau}_d} \qquad 2.33$$

where δ = Time delay mismatch = $(\tau_d - \hat{\tau}_d)$

Substituting $s = j\omega$, the magnitude ratio of the corresponding frequency response transfer function $G_{OL}(j\omega)$, is obtained as

$$| G_{OL}(j\omega) | = \sqrt{2} * \sqrt{(1 - \cos(\omega \delta))} \qquad 2.34$$

value of $(\omega\delta)$.

With the addition of the filter F(s), equation 2.33 becomes

$$G_{OL}(s) = F(s)\,(e^{-\delta s} - 1)\,e^{-\hat{\tau}_d s} \qquad 2.35$$

Usually F(s) is a low pass filter which acts to stabilize the system at the expense of decreasing the system's ability to compensate for high frequency gains. If F(s) is chosen to be a simple first order lag transfer function

$$F(s) = \frac{1}{\tau_f s + 1} \qquad 2.36$$

then the magnitude ratio of the complex transfer function corresponding to equation 2.35 is obtained as

$$|G_{OL}(j\omega)| = \frac{\sqrt{2} * \sqrt{(1 - \cos(\omega\delta)}}{\sqrt{\omega^2\tau_f^2 + 1}} \qquad 2.37$$

The addition of the filter, as seen by comparing equations 2.34 and 2.37, improves the margin of stability of the control system in the presence of a time delay mismatch.

The following example compares the performance of the Smith predictor control system shown in Figure 2.3 with the inferential control system shown in Figure 2.4 in the presence of time delay mismatch.

Example 2.2

The closed loop transfer function relating the output y to the actual disturbances η for the Smith predictor configuration given in Figure 2.3 is

$$\frac{y(s)}{\eta(s)} = \frac{[\ 1\ + G_c(s)\ G_p^*(s) - G_c(s)\hat{G}_p(s)\ ]}{[1 + G_c(s)G_p^*(s) + G_c(s)\ (\ G_p(s) - \hat{G}_p(s)\ )]} \qquad 2.38$$

while the closed loop transfer function for the inferential control system presented in Figure 2.4 is given by

$$\frac{y(s)}{\eta(s)} = [1 - G_p(s)G_I(s)T'(s)].\eta(s) \qquad 2.39$$

where $$T'(s) = \frac{F(s)}{[\ 1 + F(s)D(s)G_I(s)]} \qquad 2.40$$

$$D(s) = (G_p(s) - \hat{G}_p(s)) \qquad 2.41$$

Consider the following process

$$G_p(s) = \frac{1}{(48s + 1)}\ e^{-30\ s} \qquad 2.42$$

The process model is assumed to be

$$\hat{G}_p(s) = \frac{1}{(48s + 1)}\ e^{-(30\ +\ \delta)s} \qquad 2.43$$

where δ represents the time delay mismatch

The controller $G_c(s)$ for the Smith predictor control scheme

is chosen as an ideal PI controller i.e the controller is given by

$$G_c(s) = K_p + (K_i / s)$$

The controller parameters estimated using the Cohen-Coon (C-C) method (Seborg et al., 1989), with δ assumed to be zero are found to be $K_p = 2$, $K_i = 0.1\ s^{-1}$.

The inferential controller $G_I(s)$ is chosen to be the inverse of the process model

i.e $$G_I(s) = (\tau_p s + 1) = (48\ s + 1) \qquad 2.44$$

and the filter F(s), for the inferential control system, is chosen as

$$F(s) = \frac{1}{\delta s + 1} \qquad 2.45$$

The time delay is expected to vary by 0 to 20 seconds. Figure 2.5 shows the response of the two different control systems to a unit step increase in disturbance η. From Figures 2.5, it can be seen that the performance of both systems deteriorate in the presence of a time delay mismatch. It is also seen by comparing the mean absolute errors, for the same magnitude of mismatch in time delays, that the inferential control system performs better than the Smith predictor strategy.

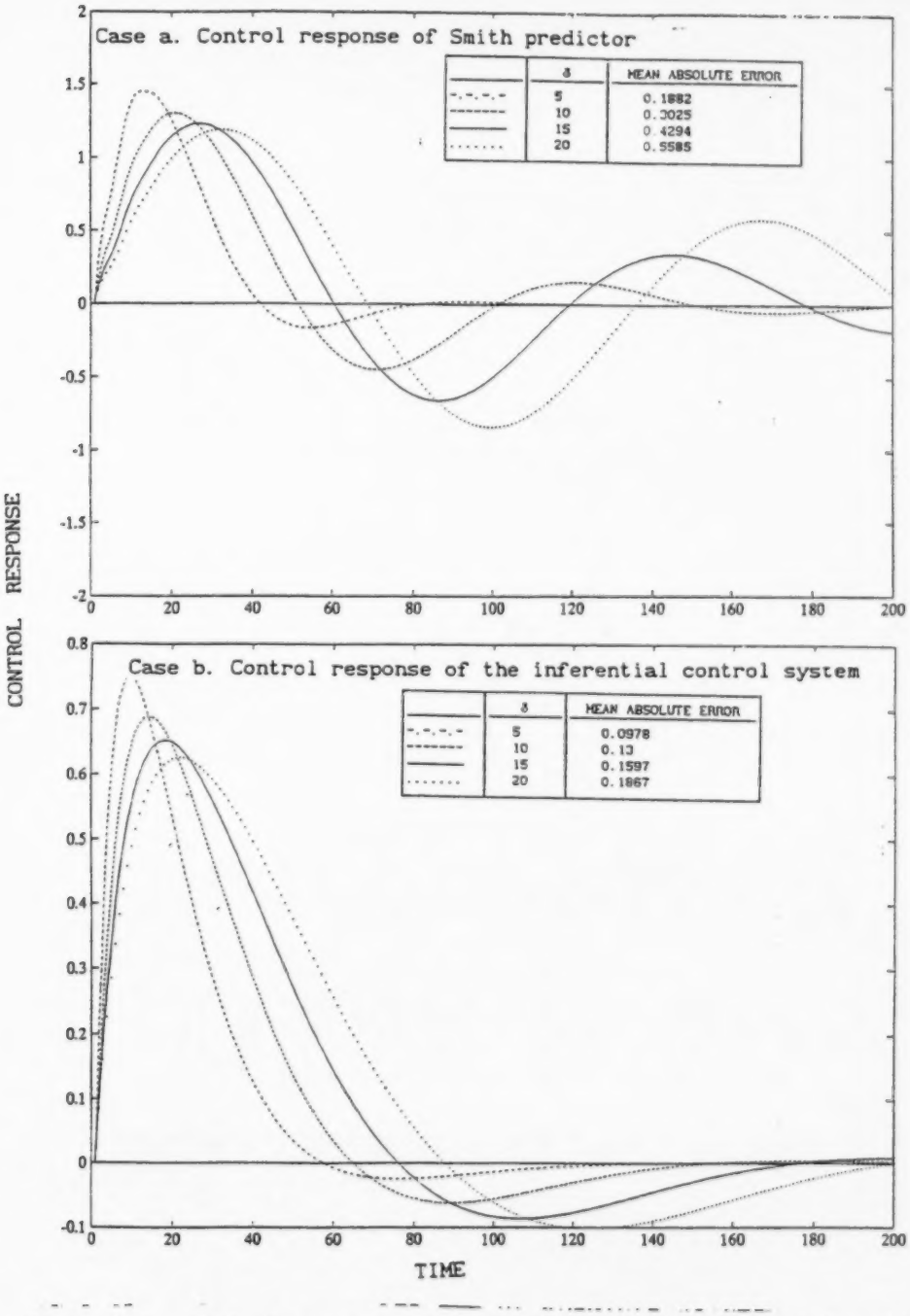

Figure 2.5 Control response of the Smith predictor and the inferential control systems to a unit step disturbance

2.4.5 Effect of an error in the process gain

Recall that the characteristic open loop transfer function is given by

$$G_{OL}(s) = F(s)[G_p(s) - \hat{G}_p(s)]\, G_I(s) \qquad 2.46$$

Consider the case of the process

$$G_p(s) = \frac{K\ e^{-\tau_d s}}{(\tau_p s + 1)} \qquad 2.47$$

and the process model is considered to be

$$\hat{G}_p(s) = \frac{\hat{K}\ e^{-\tau_d s}}{(\tau_p s + 1)} \qquad 2.48$$

with the controller given by

$$G_I(s) = \frac{1}{\hat{K}} (\tau_p s + 1) \qquad 2.49$$

If F(s) is taken as unity, substituting $s=j\omega$, the magnitude of the frequency response function is

$$|G_{OL}(j\omega)| = \left| \frac{(K - \hat{K})}{\hat{K}} \right| \qquad 2.50$$

For stability, the magnitude of the frequency response function of the open loop transfer function must be less than unity. However, in order to meet recommended practice (Seborg et al., 1989), the magnitude must be less than 0.3 at all frequencies.

i.e $$\left| \frac{K - \hat{K}}{\hat{K}} \right| < 0.3 \qquad 2.51$$

so that model-plant mismatch of 30 % in the estimation of the process gain cannot be tolerated if a filter is not used.

On the other hand, with the addition of a filter given by

$$F(s) = \frac{1}{(\tau_f s + 1)}$$

where, for arguments sake, if we assume the filter time constant to be equal to the time delay, i.e $\tau_f = \tau_d$, then the magnitude and the phase angle of the open loop frequency response function will be given by

$$|G_{OL}(j\omega)| = \frac{[(K - \hat{K})/\hat{K}]}{\sqrt{(\omega^2 \tau_d^2 + 1)}} \qquad 2.52$$

$$\angle G_{OL}(j\omega) = -\omega \tau_d - \tan^{-1}[1/(\omega\tau_f)]$$

$$= -\omega \tau_d - \tan^{-1}[1/(\omega\tau_d)] \qquad 2.53$$

At the critical frequency, equating the phase angle given by equation 2.53 to $-\Pi$ radians , the inequality corresponding to equation 2.51, for the magnitude ratio given by equation 2.52 becomes

$$\left| \frac{K - \hat{K}}{\hat{K}} \right| < 0.9 \qquad 2.54$$

Thus , with the addition of the filter, the control system can now tolerate a model-plant mismatch of 90 % in the estimation of the process gain.

2.4.6 Effect of error in process time constant

From the frequency response function given by equation 2.49, with a unity filter, for an error in the process time constant, at higher frequencies ($\omega\tau_p >> 1$), the magnitude ratio is

$$|G_{OL}(j\omega)| = \left| \frac{(\hat{\tau}_p - \tau_p)}{\tau_p} \right| \qquad 2.55$$

and the condition for acceptable stability now becomes,

$$\left| \frac{(\hat{\tau}_p - \tau_p)}{\tau_p} \right| < 0.3 \qquad 2.56$$

Just as for the case of an error in process gain, it can be similarly illustrated that the stability of the system response can be improved significantly by including a filter. A suitable choice for the filter is to select F(s) as a first order lag with a time constant on the order of $(\hat{\tau}_p - \tau_p)$ (Brosilow, 1979).

2.4.7 Smith predictor with disturbance filter

Performance of the Smith predictor control system deteriorates in the presence of model-plant mismatch (MPM) as was shown by Example 2.2. However, rather than detuning the controller to improve the performance in the presence of model-plant

mismatch, use of a low pass disturbance filter would be a better alternative. Furthermore by locating the filter in the feedback path of the disturbance, as shown in Figure 2.6, the tracking properties of the control system will not be affected by the presence of the filter. The filter time constant becomes an additional tuning parameter which can be adjusted to arrive at a desirable compromise between the robustness of the control system and sensitivity to the presence of unmeasured disturbances.

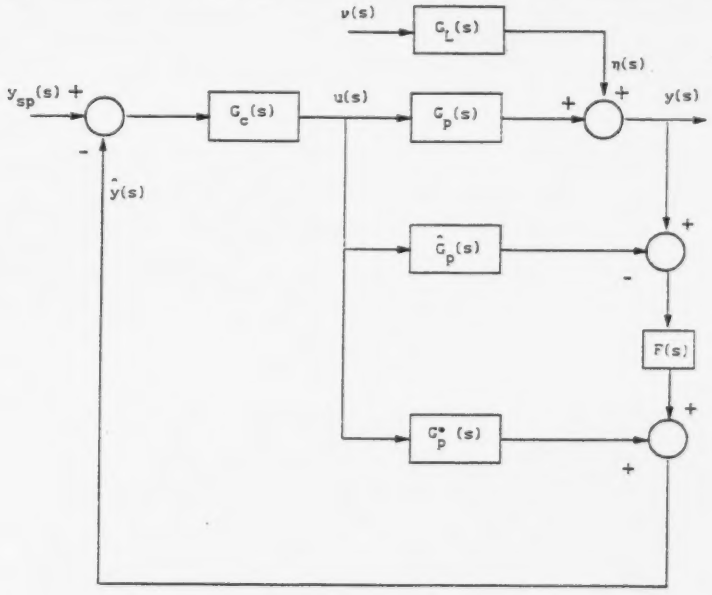

Figure 2.6 Block diagram of a Smith predictor control system utilizing a disturbance filter

From Figure 2.6, the closed loop transfer function between the output y and the disturbance η can be shown to be

$$\frac{y(s)}{\eta(s)} = \frac{[\,1 + G_c(s)\,G_p^*(s) - G_c(s)\,\hat{G}_p(s)\,F(s)\,]}{[\,1 + G_c(s)\,G_p^*(s) + F(s)\,G_c(s)\,D(s)\,]} \qquad 2.57$$

which in the absence of model-plant mismatch becomes

$$\frac{y(s)}{\eta(s)} = \frac{[\ 1 + G_c(s)\ G_P^*(s) - G_c(s)\ \hat{G}_p(s)\ F(s)\ ]}{[\ 1 + G_c(s)\ G_p^*(s)\ ]} \qquad 2.58$$

Comparing equation 2.58 to equation 2.38, we can see that a low pass filter F(s), in addition to the controller $G_c(s)$ would tend to stabilise the sytem in the presence of high frequency modelling errors.

2.5 Application of a Smith predictor control strategy in conjunction with inferential estimation to control processes with unmeasured outputs

A Smith predictor control scheme configuration used in conjunction with suboptimal inferential estimation, as shown in Figure 2.7, can be used to provide inferential control. The difference between the control system shown in Figure 2.7, henceforth referred to as the Smith inferential controller and the inferential control system of Figure 2.1 is that the Smith inferential control system is a feedback control design while the original inferential control system was basically a feedforward control strategy. The advantage is that the design of the Smith inferential controller $G_c(s)$ is simpler than the design of the model inverse controller $G_I(s)$.

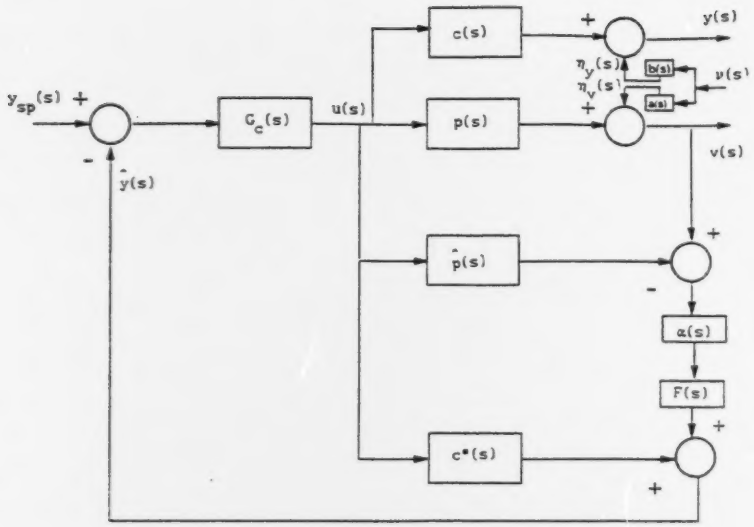

Figure 2.7 Smith predictor control system in conjunction with inferential estimation

From Figure 2.7, the transfer function relating the input u to the unmeasured disturbance ν can be shown to be

$$\frac{u(s)}{\nu(s)} = \frac{-[G_c(s)\,\alpha(s)a(s)\,F(s)\,]}{[\,1 + G_c(s)\,c^*(s) + \alpha(s)G_c(s)F(s)D(s)\,]} \qquad 2.63$$

$$D(s) = [\,p(s) - \hat{p}(s)] \qquad 2.64$$

If there is no model-plant mismatch, i.e D(s)= 0, the closed loop transfer function between the primary input y and the unmeasured disturbance ν is

$$y(s) = b(s)\,\nu(s) + c(s)\,u(s)$$

$$= \frac{[\, b(s) + b(s)G_c(s)c^*(s) - c(s)G_c(s).\alpha(s).a(s).F(s)\,]}{[\, 1 + G_c(s)\,c^*(s)\,]}\,\nu(s)$$

Since the estimator is designed to minimize the estimation error (c.f. equation 2.20) the above expression can be further simplified by making the assumption of perfect estimation (i.e $b(s)= \alpha(s)a(s)$) to yield

$$y(s) = \frac{[\, 1 + G_c(s)c^*(s) - G_c(s)c(s)F(s)\,]}{[\, 1 + G_c(s)\,c^*(s)\,]}\,b(s)\,\nu(s) \qquad 2.65$$

which on furthur simplification yields

$$y(s) = \frac{[\, 1 + G_c(s)c^*(s) - G_c(s)c(s)F(s)\,]}{[\, 1 + G_c(s)\,c^*(s)\,]}\,\eta_y(s) \qquad 2.66$$

Comparing equations 2.66 and 2.39, we observe that the closed loop properties of the Smith inferential control system are similar to those of the Smith predictor in the absence of model-plant mismatch and assuming perfect estimation.

The continuous time block diagram of the Smith inferential control system can be redrawn for discrete time as shown in Figure 2.8. It follows from this representation that the prediction for the primary output is given by

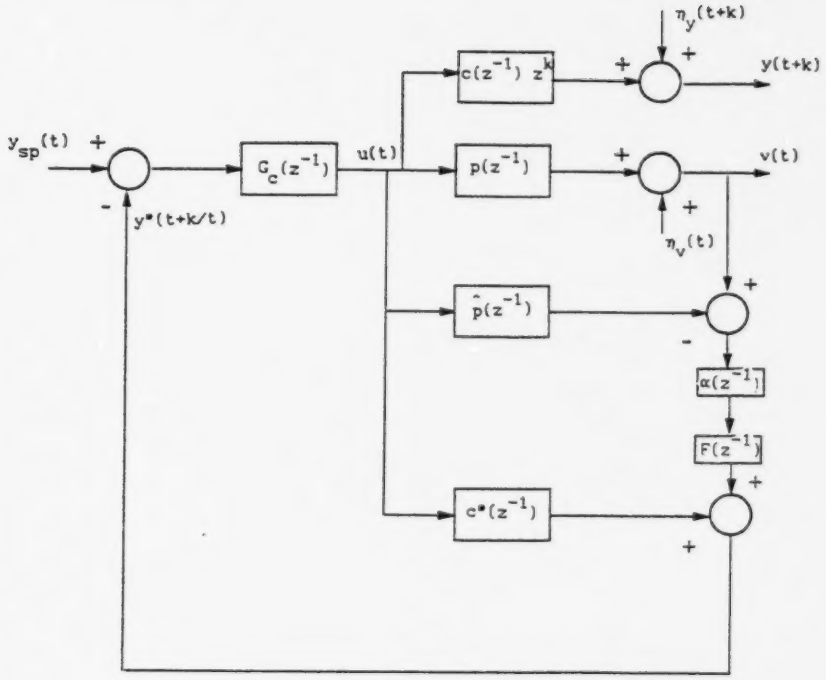

Figure 2.8 Discrete time block diagram representation of the predictive Smith inferential control scheme

$$y^*(t+k/\ t) = c^*(z^{-1})\ u(t) + F(z^{-1})\ \alpha(z^{-1}\ )\ [v(t)\ - \hat{p}(z^{-1})\ u(t)] \qquad 2.67$$

In the absence of model-plant mismatch and in the absence of unmeasured disturbances, the prediction is simply the undelayed output of the process.

2.6 An adaptive inferential control algorithm for chemical processes with intermittent measurements

Shen and Lee (1988) retained the structure of the inferential control system originally proposed by Brosilow and Tong (1978 b), but proposed the use of an adaptive inverse controller which was a stable approximation to the inverse of the process model. The inverse controller, also referred to as a deconvolution controller, could be updated directly from the measurements of the input and output variables.

This control strategy was later applied to the problem of controlling processes in which measurements of the controlled output were obtained at a slow, infrequent rate. This section is devoted entirely to the study of this adaptive inferential control algorithm developed by Shen and Lee (1989) for chemical processes with intermittent measurements.

The block diagram of the adaptive inferential control system is shown in Figure 2.9. The secondary process model $\hat{p}(z^{-1})$ is updated at a regular sampling time using frequent measurements of the secondary output. On the other hand, the primary process model $\hat{c}(z^{-1})$ is updated at a smaller sampling frequency due to the intermittent nature of the primary output measurements. This model is then used for the adaptation of the controller $G_I(z^{-1})$.

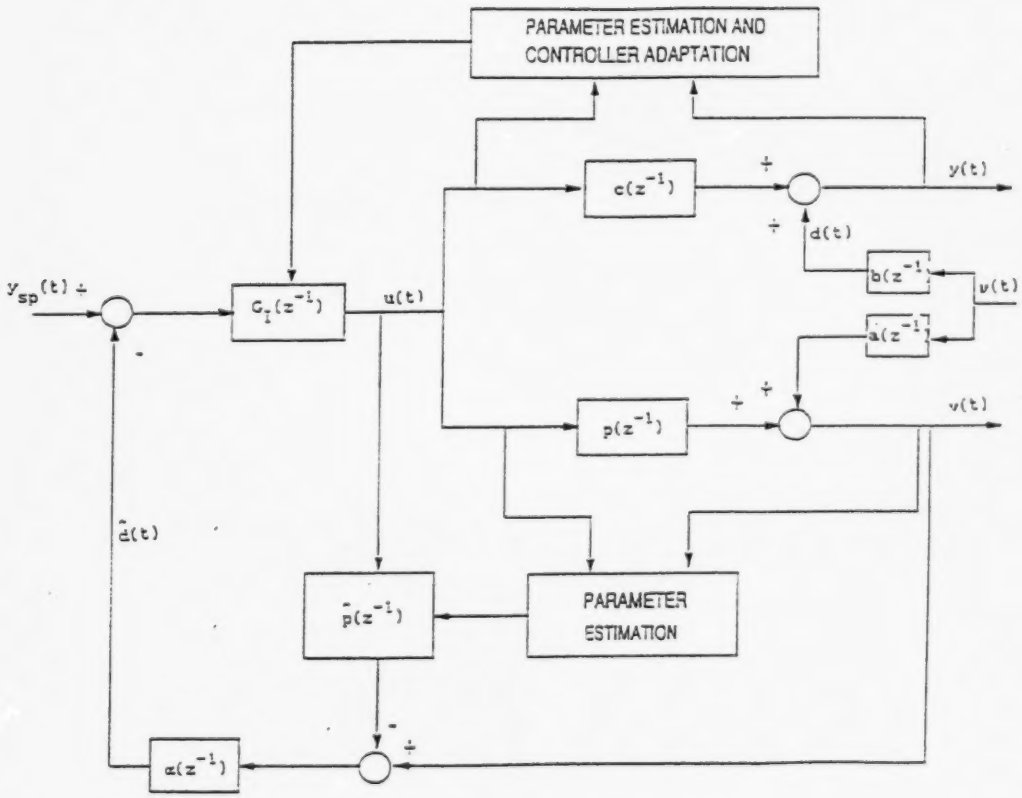

Figure 2.9 Block diagram representation for the adaptive inferential control strategy of Shen and Lee (1989)

2.6.1 Process model

Process models are required to estimate the process outputs and to design the inferential controllers. The process model that is employed is a general transfer function model, expressed as the difference equation

$$y(t) = \sum_{1}^{n} a_i \, y(t-i) + \sum_{1}^{n} b_i \, u(t-k-i) + d \qquad 2.68$$

It should be noted that the bias term, d, has been included so that erroneous model parameters would not be estimated when unmeasured load disturbances are present.

Due to the intermittent nature of the measurements of the primary output, use of this transfer function model would be unsatisfactory. In order to explain this, let us assume that the primary process output is available every JT^{th} sample period, where T is the basic sampling period, then a simple first order model for the primary process would be

$$y(t) = a_1 \, y(t-(JT)) + b_1 \, u(t-(JT)) + d \qquad 2.69$$

However, since measurements of the secondary process output are available at a frequent sampling rate, control action is adjusted equally fast to accommodate the process changes and consequently result in efficient control. As a result, any change in the control action occurring between t and t-JT sampling

instants will not be accounted for when equation 2.69 is updated. To overcome this difficulty, all the inputs between the t and t-JT sampling instants are incorporated in the process model. So equation 2.69 becomes

$$y(t) = a_1\, y(t-(JT)) + b_1\, u(t-1) + \ldots + b_J\, u(t-(JT)) + d \qquad 2.70$$

It should be noted that equation 2.70 is a nonminimal representation of a first order process. The asymptotic stability and parameter convergence of such nonminmal model based output predictors has been formally proved by Lu (1990).

2.6.2 Deconvolution controller

Inferential control systems typically utilize a stable approximation to the inverse of a process model but Shen and Lee (1988) have used a different approach than previously discussed controller design methods. Their approach involves the design of an adaptive inverse controller. The design of the adaptive inverse controller is based on the assumption that a reasonably good process model can be obtained by using on-line parameter estimation techniques. In their scheme, the adaptive inverse controller uses the process output as an input to the discrete convolution model as

$$u(t-k) = \sum_{i}^{s} y^{*}(t-i)\, g(i) \qquad 2.71$$

with the weights g(i) adapted recursively to cause its output to be a least squares fit to the process input. To obtain and update the deconvolution model, the process model output with the bias term being excluded is used instead of the actual process output to prevent biasing of the controller parameters. Adaptation of the controller parameters is also performed only at infrequent sampling instants with the updating based on the most recently updated process model outputs excluding the bias term and to further improve the stability of the control system, a stabilizing first order filter is employed.

The filter

$$u_c(t) = (1 - \sigma_u)\, u_c(t-1) + \sigma_u\, u(t) \qquad 2.73$$

is based on the structural equivalence between the inferential control and the internal model control system (Brosilow, 1979). Also for nonminimum phase systems, since an exact inverse can lead to an unstable controller, a suitable time delay k is included in equation 2.71 to obtain a stable causal approximation of the inverse.

2.7 Conclusions

The inferential estimator proposed by Brosilow and co-workers estimates the primary output by inferring the effects of unmeasured disturbances on the primary output from secondary measurements. The relative success of the strategy depends on whether a set of such secondary measurements exists in a given realistic situation.

Design of the inferential estimator is fraught with difficulties. Unmeasured inputs affecting the primary output cannot be identified in reality and even if they are identified, one cannot model their effects on the primary output experimentally because they are simply not measurable. Assuming then, that either mechanistic models are used or in case mechanistic modelling is complicated (e.g modelling of distillation columns), approximate models are established using simulators,the design of the inferential estimator is still carried out by ad-hoc means and this naturally results in steady state offset in estimation and control. This motivates us to look for more direct means of estimation; that is circumventing the step of inferring the effects of unmeasured disturbances by designing an estimator which directly estimates the primary output from secondary measurements.

If the inferential estimation scheme discussed in this chapter is incorporated in a self-tuning scheme (e.g Shen and Lee controller, 1989), a problem may be easily encountered in implementing such a strategy. This problem is related to the

philosophy of combining on-line identification with an internal model scheme. For example, it would not be correct to identify the secondary process model parameters using secondary output measurements corrupted by unmeasured disturbances and simultaneously use such a model to infer the effects of unmeasured disturbances. This problem can be resolved by assuming a disturbance model or by estimating the effects of unmeasured disturbances separately by adding a bias term to the process model as was done in case of the Shen and Lee control strategy.

CHAPTER 3 STATE ESTIMATION METHODS IN INFERENTIAL CONTROL

3.1 Introduction

In the last chapter, some model based ad-hoc approaches towards inferential estimation were considered. Extensive theoretical presentations related to the estimation of unmeasured outputs or 'states' can be found in text books on linear systems. (e.g., Kailath, 1980; Franklin et al., 1990; Kwakernaak and Sivan, 1972) In this chapter, we present an overview of some of the relevant and important results on state estimation and consider some strategies in the published literature that have exploited these results to resolve some issues in inferential estimation.

3.2 State space models

A state space model of a system, is simply a collection of linear first order differential equations which completely describe the dynamic behavior of a system. For example, the system of a mass attached to a spring can be described by the following differential equation

$$\frac{d^2x}{dt^2} = -k_1 x + F_1 \qquad 3.1$$

which can be broken down into a system of first order differential equations by defining

$$x_1 = x \tag{3.2}$$

$$x_2 = \frac{dx_1}{dt} \quad ; \quad u = F_1 \tag{3.3}$$

resulting in the state space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.4}$$

State space models can also be developed for nonlinear systems by linearizing the system equations around some nominal solutions (reference states) assuming that the deviations of the states from the reference states are very small.

The main advantage of using state space models is that they lend themselves very easily to matrix algebraic manipulations which can be easily performed on digital computers.

To understand more about the states themselves, let us consider the following general form of a state space model

$$\begin{aligned} \dot{\underline{x}}(t) &= \underline{\underline{F}}\, \underline{x}(t) + \underline{\underline{G}}\, \underline{u}(t) \\ \underline{y}(t) &= \underline{\underline{h}}\, \underline{x}(t) \end{aligned} \tag{3.5}$$

where the system is broken down into n differential equations, resulting in n states and the meaningful output vector $\underline{y}$ is a linear combination of the n states.

We note that if we know the value of these states at any

given time $t = t_0$, then given the knowledge of $\underline{u}(t)$ for $t \geq t_0$ we can calculate all present and future values of the output vector $\underline{y}(t)$ and of the states themselves without any knowledge of the old history of inputs and responses. Thus, the knowledge of the state vector at any time specifies the state or the condition of the system at that time. It should also be noted that the solution of the set of differential equations is unique to the knowledge of the initial condition of the states themselves.

3.2.1 Discrete state space representation

The set of first order differential equations of equation 3.5 can be solved for any particular set of initial state values and the solution is given by (Franklin et al., 1990)

$$\underline{x}(t) = e^{\underline{\underline{F}}\,(t-t_0)}\,\underline{x}(t_0) + \int_{t_0}^{t} e^{\underline{\underline{F}}(t-t_0)}\,\underline{\underline{G}}\,\underline{u}(\tau) \qquad 3.6$$

Letting $t = iT + T$ and $t_0 = iT$, where T is the basic sampling period and i is an integer, and assuming a zero order hold, i.e assuming that $\underline{u}(\tau)$ is piecewise constant between samples, equation 3.6 reduces to difference equations in standard form

$$\begin{aligned} \underline{x}(t+1) &= \underline{\underline{\Phi}}\,\underline{x}(t) + \underline{\underline{\Gamma}}\,u(t) \\ \underline{y}(t) &= \underline{\underline{h}}\,\underline{x}(t) \end{aligned} \qquad 3.7$$

3.3 State observability

In section 3.2, we concluded that all the future values of the responses can be calculated provided the initial conditions of the states are known.

$$\begin{aligned} \dot{\underline{x}}(t) &= \underline{\underline{F}}\,\underline{x}(t) + \underline{\underline{G}}\,\underline{u}(t) \qquad t \geq 0 \\ \underline{y}(t) &= \underline{\underline{h}}\,\underline{x}(t), \quad \underline{y}(0) = \underline{y}_0 \end{aligned} \tag{3.8}$$

If the matrices { $\underline{\underline{F}}$, $\underline{\underline{G}}$, $\underline{\underline{h}}$ } and also the inputs { $\underline{u}$, $t \geq 0$ } are known then the state observability problem constitutes translating our knowledge of the initial condition $\underline{y}_0$ to determine the appropriate initial values {$\underline{x}(0)$ } so that we can obtain $\underline{x}(t)$ as a function of t {$t \geq 0$ }. From equation 3.5, since

$$\underline{y}(t) = \underline{\underline{h}}\,\underline{x}(t)$$

it follows that

$$\begin{aligned} \underline{y}^{(1)}(t) &= \underline{\underline{h}}\,\underline{x}(t) = \underline{\underline{h}}\,\underline{\underline{F}}\,\underline{x}(t) + \underline{\underline{h}}\,\underline{\underline{G}}\,\underline{u}\,(t) \\ \underline{y}^{(2)}(t) &= \underline{\underline{h}}\,\dot{\underline{x}}^{\cdot}(t) = \underline{\underline{h}}\,\underline{\underline{F}}^2\,\underline{x}(t) + \underline{\underline{h}}\,\underline{\underline{F}}\,\underline{\underline{G}}\;\underline{u}(t) + \underline{\underline{h}}\,\underline{\underline{G}}\;\dot{\underline{u}}(t) \end{aligned} \tag{3.9}$$

and so on, which can be conveniently arranged in matrix form as

$$\underline{\underline{Y}}(t) = \underline{\underline{\Theta}}\,\underline{x}(t) + \underline{\underline{T}}\,\underline{\underline{U}}(t) \tag{3.10}$$

where $\underline{\underline{Y}}(t) = [\ \underline{y}(t)\ \ \underline{y}^{(1)}(t) \ldots \ \underline{y}^{(n-1)}(t)\ ]^T$

$\underline{\underline{U}}(t) = [\ \underline{u}(t)\ \ \underline{u}^{(1)}(t) \ldots \ \underline{u}^{(n-1)}(t)\ ]^T$

$$\underline{\underline{\Theta}} = \underline{\underline{\Theta}}(\underline{h}, \underline{F}) = [\ \underline{h}^T \quad \underline{F}^T\underline{h}^T \quad \ldots \quad (\underline{F}^T)^{n-1}\ \underline{h}^T\ ]^T$$

Assuming that $\underline{U}(0-) = \underline{0}$, we then have

$$\underline{V}(0-) = \underline{\underline{\Theta}}\ \underline{x}(0-) \qquad 3.11$$

The question obviously is whether we can find an initial state vector $\underline{x}(0-)$ for a given vector $\underline{V}(0-)$. The solution lies in the theory of linear equations and the answer is that this is possible provided the n columns of the $\underline{\underline{\Theta}}$ matrix are linearly independent, i.e if the n x n $\underline{\underline{\Theta}}$ matrix is nonsingular.

When the $\underline{\underline{\Theta}}$ matrix is nonsingular, the state space realization is said to be 'observable' and the matrix $\underline{\underline{\Theta}}(\underline{h}, \underline{F})$ is called the observability matrix of the pair $\{\ \underline{h}, \underline{F}\ \}$ (Kailath, 1980).

For multivariable systems, since observabilty properties of the system are not intuitively obvious, observable canonical forms are used to describe completely observable multivariable plants. Any (observable) state space formulation can be transformed into an observable canonical state representation (Walgama, 1989). For example, a single input - two output observable canonical state space representation is used in Chapter 4 to describe the multivariable plant model.

3.3.1 State observability in inferential control schemes

The inferential control schemes considered so far are based on the inference of the effects of unmeasured disturbances on the primary output using secondary measurements. For nonstationary disturbances, it has been shown (Morari and Stephanopoulus, 1980) that the observability criterion is violated, i.e the $\underline{\underline{\Theta}}$ matrix is singular unless the number of measurements are equal to or greater than the number of disturbances, a situation that is rarely met in process control.

Observability is a key issue to be considered while designing state estimators to infer values of the unmeasured outputs or states from secondary measurements. As will be seen later in this chapter, some schemes (e.g., Guilandoust et al., 1985) are based on assumption of complete observability of the system from secondary measurements alone while some others (e.g., Lu, 1989) are based on less restrictive assumptions of complete system observability from both frequent secondary measurements and infrequent primary output measurements.

3.4 State estimator design

The discrete time state space plant model given in equation 3.7 with the addition of noise terms to the states and the measurements can be written as

$$\begin{aligned} \underline{x}(t+1) &= \underline{\underline{\phi}}\, \underline{x}(t) + \underline{\underline{\Gamma}}\, u(t) + \underline{\xi}(t) \\ \underline{y}(t) &= \underline{\underline{h}}\, \underline{x}(t) + \underline{\omega}_v(t) \end{aligned} \qquad 3.12$$

In this model, the process noise vector $\underline{\xi}(t)$ and the measurement noise vector $\underline{\omega}_v(t)$ are assumed to be zero mean white noise sequences with covariances or noise levels defined by

$$E\{\underline{\xi}(t)\,\underline{\xi}^T(t)\} = \underline{\underline{R}}_{\xi} \;,\; E\{\underline{\omega}_v(t)\,\underline{\omega}_v^T(t)\} = \underline{\underline{R}}_v \qquad 3.13$$

State estimation methods consider the problem of reconstructing the states of the plant model from available output measurements that are corrupted by process and measurement noise. A state estimator design may be classified as a current estimator design, if the calculation of the state estimate $\hat{\underline{x}}(t)$ is based on measurements up to and including the t^{th} instant or as a prediction estimator design, if the state estimate $\bar{\underline{x}}(t)$ is only based on measurements up to and including the $(t-1)^{th}$ sampling instant. The approach to calculation of these two types of state estimates will now be considered.

3.4.1 Prediction estimators

The prediction estimator is based on the idea that the plant model itself can be used to predict a one-step ahead estimate of the states as in

$$\bar{\underline{x}}(t+1) = \underline{\underline{\Phi}}\ \bar{\underline{x}}(t) + \underline{\underline{\Gamma}}\ \underline{u}(t) \qquad 3.14$$

The estimator is shown in Figure 3.1. The estimator is initialized by setting $\bar{\underline{x}}$ (0) equal to the true $\underline{x}(0)$. As can be seen this is an open loop estimator, because it does not utilize any measurements of the output. Consequently, we can expect the estimates to diverge from the true values, unless the dynamics of the estimator exactly match the plant dynamics. In this case, the initial error would reduce to zero at a rate governed by the dynamics of the plant.

However, if the difference between the estimated output and the measured output is fedback to the estimator, the divergence should be minimized. This would result in estimation of the states using

$$\bar{\underline{x}}(t+1) = \underline{\underline{\Phi}}\ \bar{\underline{x}}(t) + \underline{\underline{\Gamma}}\ \underline{u}(t) + \underline{\underline{L}}_p\ [\underline{y}(t) - \underline{\underline{h}}\ \bar{\underline{x}}(t)] \qquad 3.15$$

where $\underline{\underline{L}}_p$ is the feedback gain matrix

A schematic representation of this is given in Figure 3.2.

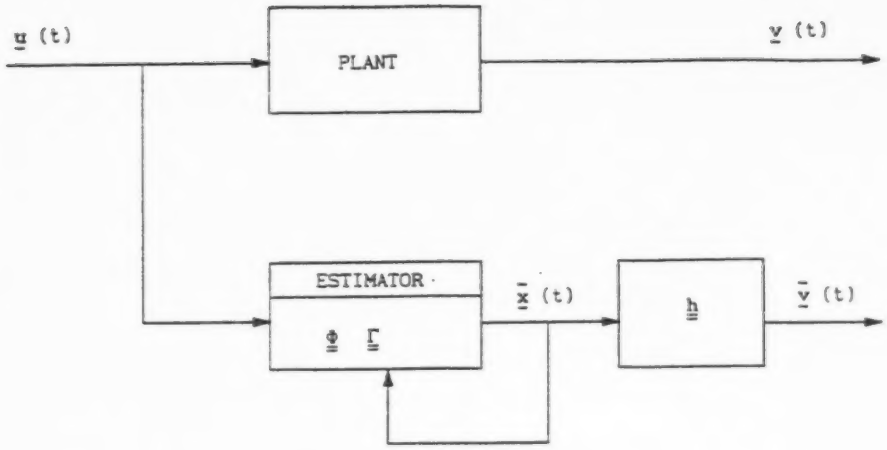

Figure 3.1 An open loop state estimator design

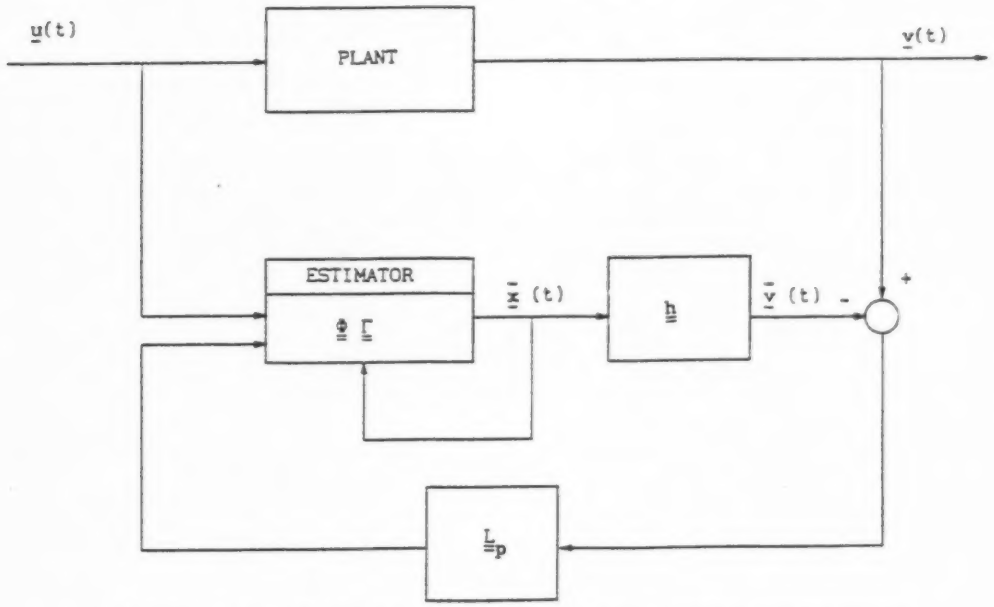

Figure 3.2 Prediction state estimator using measurement feedback

The dynamics of the estimation error in this case depends on the roots of $[\underline{\underline{\Phi}} - \underline{\underline{L}}_p \underline{\underline{h}}]$, in other words the error will converge to zero and sufficiently fast provided the elements of the estimator gain matrix $\underline{\underline{L}}_p$ are large. Roots of the estimator can be uniquely determined only if the system is observable from $\underline{y}(t)$.

3.4.2 Current estimators

In equation 3.15, the current value of the state estimates do not carry any current information, i.e $\bar{\underline{x}}(t)$ does not depend on $\underline{y}(t)$. Therefore it is useful to construct an alternative estimator formulation that provides a current estimate $\hat{\underline{x}}(t)$ based on the current measurement vector $\underline{y}(t)$. Modifying equation 3.15 accordingly, results in the following two step estimator formulation

$$\hat{\underline{x}}(t) = \bar{\underline{x}}(t) + \underline{\underline{L}}_c (\underline{y}(t) - \underline{\underline{h}}\, \bar{\underline{x}}(t))$$

$$\bar{\underline{x}}(t) = \underline{\underline{\Phi}}\, \hat{\underline{x}}(t-1) + \underline{\underline{\Gamma}}\, \underline{u}(t-1) \qquad 3.16$$

Comparison of this formulation with that in equation 3.15 shows that the estimator gain matrices are related by

$$\underline{\underline{L}}_p = \underline{\underline{\Phi}}\, \underline{\underline{L}}_c \qquad 3.17$$

3.5 Optimal state estimation

Optimal estimation methods are attractive because they allow the designer to determine many good candidate designs for the feedback gain matrix. In fact, the time varying optimal estimator solution (commonly known as the Kalman filter) is a dual of the solution to the optimal control problem (Kwakernaak and Sivan, 1972).

3.5.1 Kalman filter

For the discrete time state space representation of the plant model given in equation 3.12, the optimal estimator solution equations for a recursive least square error criterion are given by (Franklin et al., 1990)

$$\hat{\underline{x}}(t) = \bar{\underline{x}}(t) + \underline{\underline{L}}(t)\,[\underline{y}(t) - \underline{\underline{h}}\,\bar{\underline{x}}(t)] \qquad 3.18$$

$$\bar{\underline{x}}(t+1) = \underline{\underline{\Phi}}\,\hat{\underline{x}}(t) + \underline{\underline{\Gamma}}\,\underline{u}(t) \qquad 3.19$$

The change in estimate from $\hat{\underline{x}}(t)$ to $\bar{\underline{x}}(t+1)$ is called the 'time update' whereas the change in the estimate from $\bar{\underline{x}}(t)$ to $\hat{\underline{x}}(t)$ is a 'measurement update' which occurs at the fixed time t but expresses the improvement in the estimate due to the measurements $\underline{y}(t)$. For $\underline{\underline{P}}$ representing the estimate accuracy immediately after measurement and $\underline{\underline{M}}$ a propagated value of $\underline{\underline{P}}$ valid just before the measurement, the required relations can be summarized as (Walgama, 1986)

Measurement update (at measurement time)

$$\hat{\underline{x}}(t) = \bar{\underline{x}}(t) + \underline{\underline{P}}(t)\ \underline{\underline{h}}^T \underline{\underline{R}}_v^{-1}\ [\ \underline{y}(t) - \underline{\underline{h}}\ \bar{\underline{x}}(t)] \qquad 3.20$$

$$\underline{\underline{P}}\ (t) = \underline{\underline{M}}\ (t) - \underline{\underline{M}}(t)\ \underline{\underline{h}}^T\ (\underline{\underline{h}}\ \underline{\underline{M}}(t)\ \underline{\underline{h}}\ +\ R_v)^{-1}\ \underline{\underline{h}}\ \underline{\underline{M}}(t) \qquad 3.21$$

Time update (between measurements)

$$\hat{\underline{x}}\ (t)\ =\ \underline{\underline{\phi}}\ \ \bar{\underline{x}}\ (t)\ +\ \underline{\underline{\Gamma}}\ u(t)$$

and $$\underline{\underline{M}}(t+1) = \underline{\underline{\Phi}}\ \underline{\underline{P}}(t)\ \underline{\underline{\Phi}}^T + \underline{\underline{\Gamma}}\ \underline{\underline{R}}_\xi\ \underline{\underline{\Gamma}}^T \qquad 3.23$$

where the initial conditions for $\bar{\underline{x}}(0)$ and $\underline{\underline{M}}(0)$ must be known or assumed.

3.5.2 Steady state Optimal Estimation

It is often desirable to utilize a constant estimator gain matrix as it considerably simplifies the overall control system design. Many systems operate for long periods of time and can be treated mathematically as if operating for infinite time. In this case the constant gain is satisfactory because the early transient period has no significant effect. Another reason for using a constant gain Kalman filter, as will be seen later is that, once the optimal steady state Kalman gains are determined, they can be used to calculate (using innovations analysis) optimal filter transfer functions that can be easily implemented in recursive

output estimator designs. Regardless of the motivation, a constant-gain Kalman filter is identical in structure to the estimator designs expressed by equations 3.15 and 3.16, the only difference being that the gain, $\underline{\underline{L}}$, is determined so that the estimate errors are minimized for the assumed level of process and measurement noise and therefore it replaces the pole placement approach followed in the design of $\underline{\underline{L}}_c$ or $\underline{\underline{L}}_p$. Equations 3.21 and 3.23 can be solved for the steady state case and the solution for the steady state Kalman gain can be expressed as (Franklin et al., 1990)

$$\underline{\underline{L}} = \underline{\underline{M}}_{ss} \underline{\underline{h}}^T (h \underline{\underline{M}}_{ss} h^T + R_v)^{-1} \qquad 3.24$$

3.6 Kalman filter based inferential estimation

Inferential estimation refers to estimation of unmeasured primary outputs using information available from secondary output measurements. An ideal situation arises when the plant states are completely observable from the secondary outputs. In such a case, Kalman filtering techniques can be employed to estimate the plant states from the secondary output measurements. In 1985, Guilandoust (Guilandoust et al., 1985) proposed a Kalman filter based inferential estimation scheme confined to situations where the plant states are completely observable from the secondary outputs. The essence of the strategy is summarized in this

section.

Let the plant dynamics be represented by the following discrete time state space model

$$\begin{aligned} \underline{x}'(t+1) &= \underline{\underline{\phi}}'\ \underline{x}'(t) + \underline{\underline{\Gamma}}'\ \underline{u}(t) + \underline{\eta}(t) \\ \underline{y}(t) &= \underline{\underline{h}}'\ \underline{x}'(t) + \underline{\omega}_v(t) \end{aligned} \qquad 3.25$$

The disturbance input $\underline{\eta}(t)$ is considered to be a stationary exponentially correlated noise process (as such a representation does not introduce the observability problem discussed in section 3.3.1). If all elements of $\underline{\eta}(t)$ can subsequently be assumed to be stationary random sequences so that each may be represented as the response of a stable filter to a white noise input then the poles of $\underline{\eta}(t)$ can be augmented into the system matrix and the plant model rewritten as

$$\begin{aligned} \underline{x}(t+1) &= \underline{\underline{\phi}}\ \underline{x}(t) + \underline{\underline{\Gamma}}\ \underline{u}(t) + \underline{\xi}(t) \\ \underline{y}(t) &= \underline{h}\ \underline{x}(t) + \underline{\omega}_v(t) \end{aligned} \qquad 3.26$$

The state estimates $\hat{\underline{x}}(t)$ could be reconstructed from the secondary measurements using the optimal Kalman filter algorithm described by equations 3.19 through 3.24 assuming that the plant states are completely observable from the secondary outputs.

Since the basis of this inferential estimator design is that the dynamics of the primary output are completely observable from the secondary outputs, the primary output, y(t) can be assumed to

be a linear combination of these observable states, an estimate of the primary output, $\hat{y}(t)$, could then be computed from the state estimates $\hat{\underline{x}}(t)$ at every time instant.

There are two difficulties that can be identified with this estimation strategy. The first problem is related to the assumption of complete observability of the plant states from the secondary output measurements. This assumption confines the estimation algorithm to be applied to situations where the states of the primary process are completely observable from the secondary output measurements. For most plants, a set of such secondary outputs can be difficult to find or, indeed may not even exist. The second problem is related to the application of the algorithm to situations where slow measurements of the primary output are available. It is obvious that slow measurements of the primary output provide valuable information about the states of the primary process and therefore must be used to update these state variables whenever they are available. This leads us to the question as to how these measurements can be accommodated to optimally update the states of the primary process because quite obviously, this cannot be done using the present structure of the Kalman filter.

3.7 Introduction to multirate inferential control

A multirate adaptive inferential control strategy for processes in which slow or intermittent meeasurements of the primary output are available has been developed by Lu (1989). In this section, the strategy is briefly outlined and its shortcomings are discussed.

The plant is represented by the following discrete time state space model

$$\underline{x}(t+1) = \begin{bmatrix} 0 \ldots 0 & -a_1 & 0 \ldots 0 & -\bar{a}_1 \\ & \vdots & & \vdots \\ \underline{I}_{nv-1} & \vdots & & \vdots \\ & -a_{nv} & 0 & \vdots \\ 0 \ldots 0 & -a_{nv+1} & 0 \ldots 0 & -\bar{a}_{nv+1} \\ & \vdots & & \\ & & \underline{I}_{ny-1} & \\ 0 \ldots 0 & -a_n & & -\bar{a}_n \end{bmatrix} \underline{x}(t) + \begin{bmatrix} b_1 \\ \vdots \\ b_{nv} \\ \vdots \\ b_n \end{bmatrix} u(t)$$

$$+ [r_1 \ldots r_{nv} \ldots\ldots r_n]^T w(t) \qquad 3.27$$

$$\underline{x} = [\, x_1 \; x_2 \ldots x_{nv} \ldots x_n \,]^T$$

$$v(t) = x_{nv}(t) + \omega_v(t)$$

$$y(t) = h\, x_{nv}(t) + x_n(t) + \omega_y(t)$$

Equation 3.27, excluding the noise terms, can be rewritten as a SIMO structure excluding the noise terms, as follows

$$\underline{x}(t+1) = \begin{bmatrix} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \underline{\underline{A}}_{21} & \underline{\underline{B}}_{22} \end{bmatrix} \underline{x}(t) + \begin{bmatrix} \underline{\underline{B}}_{1} \\ \underline{\underline{B}}_{2} \end{bmatrix} u(t) \qquad 3.28$$

If it can be reasoned that $\underline{\underline{A}}_{12}$ and $\underline{\underline{A}}_{21}$ are zero matrices and h is set equal to zero then in that case the primary and the secondary process would be independent, with the first 'nv' states being observable from '$\underline{y}$ ' alone and the remaining 'ny' states being observable from 'y' alone.

By successive substitution of states in the plant model described by equation 3.27, Lu (1989) shows that the following model results

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t) + M(q^{-1})v(t) + C(q^{-1})\xi(t) \qquad 3.29$$

3.7.1 Multirate model formulation

Equation 3.29 cannot be used to estimate y(t) in an adaptive framework, as the identification algorithm would require measurements of the primary output at the frequent sampling rate. In order to develop a multirate model which facilitates identification of the model parameters using frequent measurements of the secondary output and control input and infrequent measurements of the primary output, the following approach is used.

It is assumed that the primary controlled output, y(t) is sampled every JT time units while the measured value of the secondary output, v(t) is available every T time units. The manipulated variable, u(t) is changed at the basic sampling time T

$$\text{Let} \quad A(q^{-1}) = \prod_{i=1}^{ny} [\ 1 - (\lambda_i q)^{-1}] \qquad 3.30$$

Multiplication of both sides of equation 3.29 by the polynomial

$$P(q^{-1}) = \prod_{i=1}^{ny} \left[\ 1 + (\lambda_i q)^{-1} + (\lambda_i q)^{-2} + \ldots + (\lambda_i q)^{2-J} + \ (\lambda_i q)^{1-J}\ \right] \qquad 3.31$$

yields, as can be shown by mathematical induction (Lu, 1989)

$$A_J(q^{-J})y(t) = B_J(q^{-1})u(t) + M_J(q^{-1})v(t) + C_J(q^{-1})\ \xi(t) \qquad 3.32$$

Equation 3.32 is referred to as a multirate model because the polynomial $A_J(q^{-J})$ is in terms of the infrequent sampling rate, i.e the polynomial is expressed in terms of q^{-J}, q^{-2J}... etc.

If a time delay, the same for the primary and secondary processes, is introduced in the plant model, equation 3.32 becomes

$$A_J(q^{-J})y(t) = B_J(q^{-1})u(t-k) + M_J(q^{-1})v(t) + C_J(q^{-1})\ \xi(t) \qquad 3.33$$

Equation 3.33, with the noise terms excluded is used for the estimation of the primary output in the estimation algorithm which is written simply as

$$\hat{y}(t) = \hat{\underline{\phi}}(t-1)\,\hat{\underline{\theta}}(t-1) \qquad 3.34$$

$$\hat{\underline{\phi}}(t) = [\ -\hat{y}(t-J)\ \ -\hat{y}(t-2J)\ \ldots\ \ -\hat{y}(t-M)$$
$$u(t-k-1)\ \ u(t-k-2)\ \ \ldots\ \ u(t-k-M)$$
$$v(t)\ \ v(t-1)\ \ v(t-2)\ \ \ldots\ \ v(t-M)\ ]$$

$$\hat{\underline{\theta}}(t) = [\ \hat{a}_{J1}\ \ \hat{a}_{J2}\ \cdots\ \hat{a}_{M}$$
$$\hat{b}_{J1}\ \ \hat{b}_{J2}\ \cdots\ \hat{b}_{JM}$$
$$\hat{m}_{J0}\ \ \hat{m}_{J1}\ \hat{m}_{J2}\cdots\ \hat{m}_{JM}]$$

At infrequent sampling instants, the estimator model parameters are identified using the present and past infrequent measurements of the primary output. However, for consistency, the estimated output at all sampling instants is used for feedback control.

This adaptive inferential estimation algorithm has been utilized for distillation column control using proportional plus integral (PI) feedback control action by Yiu (1989).

The main drawback of this multirate inferential estimation scheme is that the estimation strategy is based on an input-output relationship developed directly by substitution of states in the plant model. The multirate model derived from the input-output

relationship, as will be explained in Chapter 4 (section 4.5) cannot be used as an optimal estimator in the presence of noise effects.

Chapter 4 Adaptive Inferential Control For Processes With Multirate Sampling

4.1 Introduction

The Kalman filter provides an optimal time varying solution to the problem of estimating the states of a plant that are subjected to noisy disturbances using measurements that are also corrupted with noise. In the previous chapter, we saw that the Kalman filter could be applied to the problem of estimating unmeasurable primary process outputs from secondary output measurements provided it was assumed that the states of the primary process were observable from the secondary outputs. We concluded that a set of such secondary measurements would be difficult to find. An additional aspect that was considered was the availability of intermittent measurements of the primary output. We concluded that although these measurements provide valuable information about the states of the primary process, they cannot ordinarily be accommodated in the given structure of the Kalman filter.

In this chapter, we introduce a suboptimal multirate Kalman filter based on an optimal (periodic) multirate Kalman filter design, previously used in flight control applications (Glasson et al., 1980) to infer or extrapolate intersample values of the primary output from frequent secondary measurements. Lee and

Morari (1992) have recently extended the inferential control strategy to include outputs sampled at different rates using a similar multirate Kalman filter design. The innovations approach (Kailath, 1980) is used to develop an equivalent multirate minimum variance estimator for the primary output. The multirate minimum variance estimator is combined with an implicit or direct self-tuning controller to provide multirate adaptive inferential control for processes with intermittent measurements.

4.2 Plant Model

From the theory of systems, a SIMO system can be written in observable canonical form as

$$\begin{bmatrix} x_1(t+1) \\ \vdots \\ x_{ny}(t+1) \\ \hline x_{ny+1}(t+1) \\ \vdots \\ x_{n1}(t+1) \end{bmatrix} = \left[\begin{array}{ccc|ccc} -a_1 & & & -\bar{a}_1 & \cdots & 0 \\ \vdots & \multicolumn{2}{c|}{\underline{\underline{I}}_{ny-1}} & \vdots & & \vdots \\ -a_{ny} & \cdots & 0 & -\bar{a}_{ny} & \cdots & 0 \\ \hline -a_{ny+1} & \cdots & 0 & \bar{a}_{ny+1} & & \\ \vdots & & \vdots & \vdots & \multicolumn{2}{c}{\underline{\underline{I}}_{n-1}} \\ -a_{n1} & \cdots & 0 & -\bar{a}_{n1} & \cdots & 0 \end{array}\right] \underline{x}(t) + \begin{bmatrix} b_1 \\ \vdots \\ b_{ny} \\ \hline b_{ny+1} \\ \vdots \\ b_{n1} \end{bmatrix} u(t)$$

$$+ \left[r_1 \cdots r_{ny} \middle| r_{ny+1} \cdots r_{n1} \right] w(t) \qquad 4.1$$

If the states $x_1, \ldots, x_{ny}$ constitute the primary subsystem $\underline{x}^1$, and states $x_{ny+1}, \ldots, x_{n1}$ constitute the secondary subsystem $\underline{x}^2$, the plant model, in terms of the primary and secondary subsystems, can be written as

$$\begin{bmatrix} \underline{x}^1(t+1) \\ \hline \underline{x}^2(t+1) \end{bmatrix} = \left[\begin{array}{c|c} \underline{A}_{11} & \underline{A}_{12} \\ \hline \underline{A}_{21} & \underline{A}_{22} \end{array}\right] \begin{bmatrix} \underline{x}^1(t) \\ \hline \underline{x}^2(t) \end{bmatrix} + \begin{bmatrix} \underline{B}_1 \\ \hline \underline{B}_2 \end{bmatrix} u(t)$$

$$+ \left[\begin{array}{c|c} \underline{R}_1^T & \underline{R}_2^T \end{array}\right] w(t) \qquad 4.2$$

$$v(t) = \underset{(1xny)}{\left[\begin{array}{c|c} 0 & \underline{C}_2^T \end{array}\right]} \underset{(1xn)}{\underline{x}(t)} + \omega_v(t)$$

$$= \underset{(1xn1)}{\underline{H}_2} \underline{x}(t) + \omega_v(t) \qquad 4.3$$

$$n = n1 - ny$$

$$y(t) = \underset{(1xny)}{\left[\underline{C}_1^T \;\vdots\; 0\right]} \underset{(1xn)}{\underline{x}(t)} + \omega_y(t)$$

$$= \underset{(1xn1)}{\underline{H}_1} \underline{x}(t) + \omega_y(t) \qquad 4.4$$

4.3 Optimal multirate estimator design

The secondary measurement v(t) is available at the basic sampling period T. The primary output y(t) is sampled every JT time units.

Although the state model matrices are assumed to be basically time invariant, the dimension of the measurement vector changes periodically.

At the infrequent/major sampling instant t = JT

The measurement vector is defined as

$$\underline{z} = [\, y \,\vdots\, v \,]^T$$

$$= \begin{bmatrix} \underline{H}_1 \\ \hline \underline{H}_2 \end{bmatrix} \underline{x}\,(t) + \begin{bmatrix} \omega_y \\ \hline \omega_v \end{bmatrix}$$

$$= \underline{H}\ \underline{x}\,(t) + \underline{\omega}\,(t) \qquad 4.5$$

At all other sampling instants (t ≠ JT)

The measurement vector is defined as

$$\underline{z} = v$$

$$= \underline{H}_2\ \underline{x}\,(t) + \omega_v(t)$$

$$= \underline{H}\ \underline{x}\,(t) + \underline{\omega}\,(t) \qquad 4.6$$

The measurement sensitivity vector $\underline{H}$ and the measurement noise vector $\underline{\omega}$ are variables with (periodic) time variant dimensions, and the variation is given according to equation 4.5 and 4.6.

Similarly the dimensions of the measurement noise covariance matrix also varies periodically

$$\underline{\underline{R}}_z = \begin{bmatrix} R_y & \vdots & 0 \\ \hline 0 & \vdots & R_v \end{bmatrix} \quad ; \quad t = JT \qquad 4.7$$

$$\underline{\underline{R}}_z = R_v \quad ; \quad t \neq JT \qquad 4.8$$

The dimensions and magnitude of the Kalman filter gains derived by propagating the discrete Kalman filter covariance equation to a (periodic) steady state are also periodic. The optimal multirate Kalman filter consists of a time update and two independent measurement update mechanism

Time update

$$\begin{bmatrix} \hat{\underline{x}}^1(t/t-1) \\ \hline \hat{\underline{x}}^2(t/t-1) \end{bmatrix} = \left[\begin{array}{c|c} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \hline \underline{\underline{A}}_{21} & \underline{\underline{A}}_{22} \end{array}\right] \begin{bmatrix} \hat{\underline{x}}^1(t-1/t-1) \\ \hline \hat{\underline{x}}^2(t-1/t-1) \end{bmatrix} + \begin{bmatrix} \underline{B}_1 \\ \underline{B}_2 \end{bmatrix} u(t-1) \qquad 4.9$$

$$\hat{y}(t/t-1) = \underline{C}_1^T \hat{\underline{x}}^1 (t/t-1) \qquad 4.10$$

$$\hat{v}(t/t-1) = \underline{C}_2^T \hat{\underline{x}}^2 (t/t-1) \qquad 4.11$$

$$\underset{(n1xn1)}{\underline{\underline{P}}(t/t-1)} = \underline{\underline{A}}\, \underline{\underline{P}}(t-1/t-1)\, \underline{\underline{A}}^T + \underline{\underline{R}}_w \qquad 4.12$$

where $\underline{\underline{A}}$ is the system matrix and $\underline{\underline{R}}_w$ is the process noise covariance matrix.

Measurement update

The measurement update is performed as and when measurements are available. This leads to two independent update mechanisms, one which applies at the infrequent sampling instants when both the primary and secondary measurements are available and the other which applies at all other sampling instants (when only the secondary measurement is available).

At infrequent/major sampling instants, t = JT

$$\underset{(n1x2)}{\underline{\underline{K}}} = \underset{(n1xn1)}{\underline{\underline{P}}}(t/t-1) \quad \underset{(n1x2)}{\underline{H}^T} \quad [\underline{H}\,\underline{\underline{P}}(t/t-1)\,\underline{H}^T + \underset{(2x2)}{\underline{\underline{R}_z}}]^{-1} \qquad 4.13$$

The matrices $\underline{H}$ and $\underline{\underline{R}}_z$ in this case are given by equations 4.5 and 4.7.

The Kalman gain matrix can be written as

$$\underset{(n1x2)}{\underline{\underline{K}}} = \left[\begin{array}{c|c} \underline{K}_{11(nyx1)} & \underline{K}_{12\ (nyx1)} \\ \hline \underline{K}_{21(nx1)} & \underline{K}_{22(nx1)} \end{array}\right] \qquad 4.14$$

The infrequent or major update would be made according to

$$\begin{bmatrix} \hat{\underline{x}}^1(t/t) \\ \hat{\underline{x}}^2(t/t) \end{bmatrix} = \begin{bmatrix} \hat{\underline{x}}^1(t/t-1) \\ \hat{\underline{x}}^2(t/t-1) \end{bmatrix} + \left[\begin{array}{c|c} \underline{K}_{11(nyx1)} & \underline{K}_{12\ (nyx1)} \\ \hline \underline{K}_{21(nx1)} & \underline{K}_{22(nx1)} \end{array}\right] \begin{bmatrix} \nu_y \\ \nu_v \end{bmatrix} \qquad 4.15$$

where ν_y and ν_v are estimation errors associated with y and v respectively, given by

$$\nu_y(t) = y(t) - y(t/t-1) \qquad 4.16$$

$$\nu_v(t) = v(t) - v(t/t-1) \qquad 4.17$$

The propagated value of the error covariance matrix is calculated according to

$$\underset{(n1xn1)}{\underline{\underline{P}}}(t/t) = [\ \underline{\underline{I}} - \underline{\underline{K}}\ \underline{\underline{H}}\ ]\ \underline{\underline{P}}\ (t/t-1) \qquad 4.18$$

At all other sampling instants ($t \neq JT$)

$$\underset{(n1x1)}{\underline{K}} = \underset{(n1xn1)}{\underline{\underline{P}}}\ (t/t-1)\ \ \underset{(n1x1)}{H^T}\ \ [H\ \underline{\underline{P}}(t/t-1)\ H^T + \underset{(1x1)}{\underline{\underline{R}}_z}\ ]^{-1} \qquad 4.19$$

In this case, the matrices $\underline{H}$ and $\underline{\underline{R}}_z$ would be given by equations 4.6 and 4.8

The Kalman gain matrix in this case would be written as

$$\underset{(n1x1)}{\underline{\underline{K}}} = \left[\begin{array}{c} \underline{\underline{K}}_{12}\ (nyx1) \\ \hline \underline{\underline{K}}_{22}\ (nx1) \end{array} \right] \qquad 4.20$$

The frequent updates would be made according to

$$\left[\begin{array}{c} \hat{\underline{x}}^1(t/t) \\ \hline \hat{\underline{x}}^2(t/t) \end{array} \right] = \left[\begin{array}{c} \hat{\underline{x}}^1(t/t-1) \\ \hline \hat{\underline{x}}^2(t/t-1) \end{array} \right] + \left[\begin{array}{c} \underline{\underline{K}}_{12}\ (nyx1) \\ \hline \underline{\underline{K}}_{22}\ (nx1) \end{array} \right] [\ \nu_v(t)] \qquad 4.21$$

where ν_v is the estimation error associated with the secondary measurement and would be given by equation 4.17.

The propagated value of the error covariance update matrix is calculated according to

$$\underset{(n1xn1)}{\underline{\underline{P}}}(t/t) = [\ \underline{\underline{I}} - \underline{\underline{K}}\ \underline{H}\ ]\ \underline{\underline{P}}\ (t/t-1) \qquad 4.22$$

4.3.1 Remarks

A simple explanation for the filter structure would be that at the frequent sampling instants when the primary measurement is not available, the estimates for the states of the primary subsystem are extrapolated from estimates of the states of the secondary subsystem using the interaction between the two subsytems in the plant model, so the error covariance matrix for the primary subsystem grows with time. At the infrequent sampling instant, the state estimates of the primary subsystem are reset to more accurate values and the estimation error covariance is automatically reset to a smaller value due to the availability of the primary measurement. These reset estimates are then extrapolated in the subsequent cycle via integration until a new primary measurement is available.

It should also be noted that due to the unavailablity of the primary measurement between the major cycles, the Kalman gains associated with the estimation error of the secondary measurement grow with time. The Kalman gain values are reset, similar to the estimation error covariance when the slowly sampled measurement of the primary output is obtained and the steady state (periodic) pattern of increasing Kalman gains (those associated with the estimation error of the secondary measurement) is repeated in the subsequent cycle.

4.4 Suboptimal multirate estimator design

Innovations analysis has often been employed to transform state space filter formulations into difference equations directly relating the outputs to the inputs (e.g., Walgama, 1989). Innovations model analysis of the multirate Kalman filter is simplified if the following ad-hoc changes are made to the optimal multirate estimator solution

i. The cross Kalman gain terms $\underline{K}_{12}$ and $\underline{K}_{21}$ are set equal to zero for mathematical tractability. Consequently when the measurement update is performed, the states of the primary and secondary subsystems are reconstructed individually from the primary and secondary measurements respectively. The intrinsic interaction between the states of the two subsystems due to the nature of the plant model provides the means to extrapolate estimates for the states of the primary subsystem from those of the secondary subsystems.

ii. The solution to the (periodic) time varying multirate Kalman filter equations depends on the characteristics of the process and measurement noise. The iterative multirate Kalman filter equations were solved for a sample process (see Appendix A). It is observed that although the steady state solution to the multirate Kalman filter problem is periodically time varying, the magnitude of the variation in the Kalman gains between the major sampling instants is directly related to the ratio of the process and measurement

noise variances. For small ($\underline{\underline{R}}_w/\underline{\underline{R}}_z$) ratios, the intersample variation in the Kalman gains is small and the variation increases as the noise ratio is increased. A heuristic explanation for this is that for low values of the noise ratio, the Kalman filter relies more on the process model than on the measurements, and hence changes in the measurement pattern do not affect the Kalman gain calculation significantly. For very small values of the noise ratio ($\underline{\underline{R}}_w/\underline{\underline{R}}_z$), the Kalman gain $\underline{\underline{K}}_{22}$ can be assumed to be fairly constant at steady state. It is to be noted that $\underline{\underline{K}}_{11}$ is calculated only at the major sampling instant and converges to a steady state value that can be estimated offline.

The resulting suboptimal multirate Kalman filter has the following structure

Time update

$$\begin{bmatrix} \underline{\hat{x}}^1(t/t-1) \\ \hline \underline{\hat{x}}^2(t/t-1) \end{bmatrix} = \left[\begin{array}{c|c} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \hline \underline{\underline{A}}_{21} & \underline{\underline{A}}_{22} \end{array}\right] \begin{bmatrix} \underline{\hat{x}}^1(t-1/t-1) \\ \hline \underline{\hat{x}}^2(t-1/t-1) \end{bmatrix} + \begin{bmatrix} \underline{B}_1 \\ \underline{B}_2 \end{bmatrix} u(t-1) \qquad 4.23$$

$$\hat{y}(t/t-1) = \underline{C}_1^T \, \underline{\hat{x}}^1 (t/t-1) \qquad 4.24$$

$$\hat{v}(t/t-1) = \underline{C}_1^T \, \underline{\hat{x}}^2 (t/t-1) \qquad 4.25$$

The a-priori estimates of the states at any time t are conditional on data up to and including time (t-1)

Measurement updates

At infrequent sampling instants (t = JT)

$$\begin{bmatrix} \hat{\underline{x}}^1(t/t) \\ \hline \hat{\underline{x}}^2(t/t) \end{bmatrix} = \begin{bmatrix} \hat{\underline{x}}^1(t/t-1) \\ \hline \hat{\underline{x}}^2(t/t-1) \end{bmatrix} + \left[\begin{array}{c|c} \underline{K}_{11(nyx1)} & \underline{0}_{(nyx1)} \\ \hline \underline{0}_{(nx1)} & \underline{K}_{22(nx1)} \end{array}\right] \begin{bmatrix} \nu_y \\ \hline \nu_v \end{bmatrix} \qquad 4.26$$

where $\underline{K}_{11} = [\, k_1 \ldots k_{ny}]^T$

$\underline{K}_{22} = [k_{ny+1} \ldots k_{n1}]^T$

At all other sampling instants (t ≠ JT)

$$\begin{bmatrix} \hat{\underline{x}}^1(t/t) \\ \hline \hat{\underline{x}}^2(t/t) \end{bmatrix} = \begin{bmatrix} \hat{\underline{x}}^1(t/t-1) \\ \hline \hat{\underline{x}}^2(t/t-1) \end{bmatrix} + \begin{bmatrix} \underline{0}_{(nyx1)} \\ \hline \underline{K}_{22(nx1)} \end{bmatrix} [\, \nu_v(t)] \qquad 4.27$$

$$\hat{y}(t/t) = \underline{C}_1^T \hat{\underline{x}}^1 (t/t) \qquad 4.28$$

$$\hat{v}(t/t) = \underline{C}_2^T \hat{\underline{x}}^2 (t/t) \qquad 4.29$$

4.5 Innovations analysis

The innovations model analysis is used to transform Kalman filter algorithms into input-output models. Innovations analysis operations are performed on the basis of the concept of the innovations process (Kwakernaak and Sivan, 1972) which can be stated as follows.

" For an optimal filter, such as the multirate Kalman filter, the innovations process $[\underline{z}(t) - \hat{\underline{z}}(t/t-1)]$ is a sequence of zero-mean uncorrelated stochastic vectors with covariance matrix $\underline{\underline{R}}_z$". Consequently it can be inferred that the innovations process refers to the estimation errors ν_y and ν_v and their variances suggest that they are equivalent to the primary and secondary measurement noise ω_y, ω_v respectively.

Therefore from the innovations interpretation of the multirate Kalman filter, we can write

$$\nu_y(t) = y(t) - \hat{y}(t/t-1) = \omega_y(t) \qquad 4.30$$

$$\nu_v(t) = v(t) - \hat{v}(t/t-1) = \omega_v(t) \qquad 4.31$$

The innovation model for the primary subsystem can be rewritten as

Time update

$$\hat{\underline{x}}^1(t/t-1) = \underline{\underline{A}}_{11}\,\hat{\underline{x}}^1(t-1/t-1) + \underline{\underline{A}}_{12}\,\hat{\underline{x}}^2(t-1/t-1) + \underline{B}_1\,u(t-1) \qquad 4.32$$

$$\hat{y}(t/t-1) = \underline{C}_1^T\,\hat{\underline{x}}^1(t/t-1) \qquad 4.33$$

Assuming that the most current measurement of the primary output was obtained at time t, the last measurement of the primary output was then obtained at time (t-J). Therefore all the measurement updates for the states of the primary subsystem until the time (t-J) are set equal to the previous time update according to equation 4.27 i.e

A-posteriori estimate

$$\hat{\underline{x}}^1(t-i/t-i) = \hat{\underline{x}}^1(t-i/t-i-1) \quad \text{where } i = 1,\ldots,(J-1) \qquad 4.34$$

$$\hat{y}(t-i/t-i) = \underline{C}_1^T\,\hat{\underline{x}}^1(t-i/t-i) \qquad 4.35$$

The development begins by writing

$$y(t) = \underline{C}_1^T\,\hat{\underline{x}}^1(t/t-1) + \omega_y(t) \qquad 4.36$$

which on substituting for $\hat{\underline{x}}^1(t/t-1)$ from equation 4.32 yields

$$y(t) = \underline{C}_1^T \underline{\underline{A}}_{11} \hat{\underline{x}}^1(t-1/t-1) + \underline{C}_1^T \underline{\underline{A}}_{12} \hat{\underline{x}}^2(t-1/t-1) + \underline{C}_1^T \underline{B}_1 u(t-1) + \omega_y(t) \qquad 4.37$$

Since measurements of the primary output are not available at the basic sampling period, the measurement update expression at times (t-1),..., (t-J+1) is given by equation 4.34. Substituting for $\hat{\underline{x}}^1(t-1/t-1)$ from equation 4.34, we get

$$y(t) = \underline{C}_1^T \underline{\underline{A}}_{11} \hat{\underline{x}}^1(t-1/t-2) + \underline{C}_1^T \underline{\underline{A}}_{12} \hat{\underline{x}}^2(t-1/t-1) + \underline{C}_1^T \underline{B}_1 u(t-1) + \omega_y(t) \qquad 4.38$$

which applying the time update mechanism again results in

$$y(t) = \underline{C}_1^T \underline{\underline{A}}_{11} [\underline{\underline{A}}_{11} \hat{\underline{x}}^1(t-2/t-2) + \underline{\underline{A}}_{12} \hat{\underline{x}}^2(t-2/t-2) + \underline{B}_1 u(t-2)] +$$
$$\underline{C}_1^T \underline{\underline{A}}_{12} \hat{\underline{x}}^2(t-1/t-1) + \underline{C}_1^T \underline{B}_1 u(t-1) + \omega_y(t) \qquad 4.39$$

Repeating the above procedure until the sampling instant (t-J), we get

$$y(t) = \underline{C}_1^T \underline{\underline{A}}_{11}^J \hat{\underline{x}}^1(t-J/t-J) +$$
$$\underline{C}_1^T [\underline{\underline{I}} + \underline{\underline{A}}_{11} q^{-1} + \underline{\underline{A}}_{11}^2 q^{-2} + .. + \underline{\underline{A}}_{11}^{(J-1)} q^{-(J-1)}] [\underline{\underline{A}}_{12} \hat{\underline{x}}^2(t-1/t-1)] +$$
$$\underline{C}_1^T [\underline{\underline{I}} + \underline{\underline{A}}_{11} q^{-1} + \underline{\underline{A}}_{11}^2 q^{-2} + .. + \underline{\underline{A}}_{11}^{(J-1)} q^{-(J-1)}] [\underline{B}_1 u(t-1)] + \omega_y(t) \qquad 4.40$$

Since the last primary output measurement was available at sampling instant (t-J), it follows from equation 4.26 that

$$\hat{\underline{x}}^1(t-J/t-J) = \hat{\underline{x}}^1(t-J/t-J-1) + \underline{K}_{11} \omega_y(t-J)$$

and substituting for $\hat{\underline{x}}^1(t-J/t-J)$ in equation 4.40 yields

$$y(t) = \underline{C}_1^T \underline{A}_{11}^J \hat{\underline{x}}^1(t-J/t-J-1) + \underline{C}_1^T \underline{A}_{11}^J \underline{K}_{11}\omega_y(t-J) +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11}q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)}q^{-(J-1)}] [\underline{A}_{12} \hat{\underline{x}}^2(t-1/t-1)] +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11}q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)}q^{-(J-1)}] [\underline{B}_1 u(t-1)] + \omega_y(t) \quad 4.41$$

From equation 4.33 we have

$$\hat{y}(t-J/t-J-1) = \underline{C}_1^T \hat{\underline{x}}^1(t-J/t-J-1)$$

So, multiplication on both sides by $\underline{C}_1$ gives

$$\underline{C}_1 \hat{y}(t-J/t-J-1) = [\underline{C}_1 \underline{C}_1^T] \hat{\underline{x}}^1(t-J/t-J-1) \quad 4.42$$

and since $[\underline{C}_1\underline{C}_1^T]$ is invertible, we can write

$$\hat{\underline{x}}^1(t-J/t-J-1) = \underline{C}_1^+ \hat{y}(t-J/t-J-1) \quad 4.43$$

where $\underline{C}_1^+ = [\underline{C}_1 \underline{C}_1^T]^{-1}\underline{C}_1$ is a pseudo-inverse of $\underline{C}_1$

Substitution for $\hat{\underline{x}}^1(t-J/t-J-1)$ in equation 4.43 results in

$$y(t) = \underline{C}_1^T \underline{A}_{11}^J \underline{C}_1^+ \hat{y}(t-J/t-J-1) + \underline{C}_1^T \underline{A}_{11}^J \underline{K}_{11}\omega_y(t-J) +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11}q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)}q^{-(J-1)}] [\underline{A}_{22} \hat{\underline{x}}^2(t-1/t-1)] +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11}q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)}q^{-(J-1)}] [\underline{B}_1 u(t-1)] + \omega_y(t) \quad 4.44$$

We also note from equation 4.30 that

$$y(t-J) = \hat{y}\,(t-J/t-J-1) + \omega_y(t-J)$$

and on substituting for $\hat{y}(t-J/t-J-1)$ in equation 4.44, we get

$$y(t) = \underline{C}_1^T \underline{A}_{11}^J \underline{C}_1^+ y(t-J) + \underline{C}_1^T \underline{A}_{11}^J \underline{C}_1^+ \omega_y(t-J) + \underline{C}_1^T \underline{A}_{11}^J \underline{K}_{11} \omega_y(t-J) +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] [\underline{A}_{12} \hat{\underline{x}}^2(t-1/t-1)] +$$

$$\underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] [\underline{B}_1 u(t-1)] + \omega_y(t) \qquad 4.45$$

which can be rewritten as

$$\{1 - [\underline{C}_1^T \underline{A}_{11}^J \underline{C}_1^+] q^{-J}\} y(t) = \{1 + \underline{C}_1^T \underline{A}_{11}^J [\underline{C}_1^+ + \underline{K}_{11}] q^{-J}\} \omega_y(t) +$$

$$\{\underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] \underline{A}_{12} \underline{C}_2^+\} \hat{v}(t-1/t-1) +$$

$$\{\underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] \underline{B}_1\} u(t-1) \qquad 4.46$$

Equation 4.46 can be expressed in an input-output relationship as

$$A_J(q^{-J})\, y(t) = B_J(q^{-1})\, u(t-1) + M_J(q^{-1})\, \hat{v}(t-1/t-1) + C_J(q^{-J})\, \omega_y(t) \qquad 4.47$$

where

$$A_J(q^{-J}) = 1 - [\underline{C}_1^T \underline{A}_{11}^J \underline{C}_1^+] q^{-J}$$

$$B_J(q^{-1}) = \underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] \underline{B}_1$$

$$M_J(q^{-1}) = \underline{C}_1^T [\underline{I} + \underline{A}_{11} q^{-1} + \underline{A}_{11}^2 q^{-2} + .. + \underline{A}_{11}^{(J-1)} q^{-(J-1)}] \underline{A}_{12} \underline{C}_2^+$$

$$C_J(q^{-J}) = 1 + \underline{C}_1^T \underline{A}_{11}^J [\underline{C}_1^+ + \underline{K}_{11}]\, q^{-J}$$

For the analysis of the filter for the secondary subsystem, it is simpler to adopt a more direct approach. The innovations model for the secondary subsystem can be written as

Time update

$$\begin{bmatrix} \hat{x}_{ny+1}(t/t-1) \\ \vdots \\ \hat{x}_{n1}(t/t-1) \end{bmatrix} = \begin{bmatrix} -a_{ny+1} & \cdots & 0 \\ \vdots & & \\ -a_{n1} & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(t-1/t-1) \\ \vdots \\ x_{ny}(t-1/t-1) \end{bmatrix} +$$

$$\begin{bmatrix} -\bar{a}_{ny+1} & \cdots & 0 \\ \vdots & & \\ -\bar{a}_{n1} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_{ny+1}(t-1/t-1) \\ \vdots \\ \hat{x}_{n1}(t-1/t-1) \end{bmatrix} + \begin{bmatrix} b_{ny+1} \\ \vdots \\ b_{n1} \end{bmatrix} u(t-1) \qquad 4.48$$

$$\hat{v}(t/t-1) = \hat{x}_{ny+1}(t/t-1) \qquad 4.49$$

$$\hat{y}(t/t-1) = \hat{x}_1(t/t-1) \qquad 4.50$$

Measurement update

$$\begin{bmatrix} \hat{x}_{ny+1}(t/t) \\ \vdots \\ \hat{x}_{n1}(t/t) \end{bmatrix} = \begin{bmatrix} \hat{x}_{ny+1}(t/t-1) \\ \vdots \\ \hat{x}_{n1}(t/t-1) \end{bmatrix} + \begin{bmatrix} k_{ny+1} \\ \vdots \\ k_{n1} \end{bmatrix} \omega_v(t) \qquad 4.51$$

$$\hat{v}(t/t) = \hat{x}_{ny+1}(t/t) \qquad 4.52$$

$$y(t/t) = \hat{x}_1(t/t) \qquad 4.53$$

The development begins by writing

$$\hat{x}_{ny+1}(t/t) = x_{ny+1}(t/t-1) + k_{ny+1}\,\omega_v(t) \qquad 4.54$$

and substituting for $\hat{x}_{ny+1}(t/t-1)$ from the time update equation 4.48, we have

$$\hat{x}_{ny+1}(t/t) = -\bar{a}_{ny+1}\,\hat{x}_{ny+1}(t-1/t-1) + \hat{x}_{ny+2}(t-1/t-1)$$

$$- a_{ny+1}\,\hat{x}_1(t-1/t-1) + b_{ny+1}\,u(t-1) + k_{ny+1}\omega_v(t) \qquad 4.55$$

Successive substitution of states of the secondary subsystem using the time and measurement update equations 4.48 and 4.51 respectively yields

$$\hat{x}_{ny+1}(t/t) = [-\bar{a}_{ny+1}q^{-1} - \bar{a}_{ny+2}\,q^{-2} \ldots \bar{a}_{n1}q^{-n}\,]\,\hat{x}_{ny+1}(t/t) +$$

$$[-a_{ny+1}q^{-1} - a_{ny+2}\,q^{-2} \ldots a_{n1}q^{-n}\,]\,\hat{x}_1(t/t)$$

$$+$$

$$[\,b_{ny+1} + b_{ny+2}q^{-1} + .. b_{n1}\,q^{-(n-1)}\,]\,u(t-1) + [\,k_{ny+2}\,q^{-1} + \ldots k_{n1}q^{-n}\,]\,\omega_v(t)$$

$$4.56$$

This expression can be rewritten as

$$\bar{A}(q^{-1})\ \hat{x}_{ny+1}(t/t) = \bar{M}(q^{-1})\hat{y}(t-1/t-1) + \bar{B}(q^{-1})\ u(t-1) + K_2(\bar{q}^{1}\ )\ \omega_v(t) \qquad 4.57$$

where $K_2(q^{-1}) = [k_{ny+1}q^{-1} + \ldots + k_{n1}\ q^{-n}]$

which can be further simplified and written as

$$\bar{A}(q^{-1})\ v(t) = \bar{M}(q^{-1})\hat{y}(t-1/t-1) + \bar{B}(q^{-1}\ )\ u(t-1) + \bar{C}\ (q^{-1})\ \omega_v(t) \qquad 4.58$$

where $\bar{C}(q^{-1}) = \bar{A}(q^{-1})(1 - k_{ny+1}) + K_2(q^{-1})$ 4.59

In summary, the required input-output relations are

$$A_J(q^{-J})\ y(t) = B_J(q^{-1})\ u(t-1) + M_J(q^{-1})\ \hat{v}(t-1) + C_J(q^{-J})\ \omega_y(t) \qquad 4.60$$

$$\bar{A}(q^{-1})\ v(t) = \bar{B}(q^{-1}\ )\ u(t-1) + M(q^{-1})\ \hat{y}(t-1) + \bar{C}\ (q^{-1})\ \omega_v(t) \qquad 4.61$$

4.6 Multirate formulation for a simple first order process

Consider a deterministic plant model where the primary and secondary processes are assumed to be first order processes

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} a_1 & \bar{a}_1 \\ a_2 & \bar{a}_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t-k+1) \qquad 4.62$$

$$y(t) = x_1(t) \qquad 4.63$$

$$v(t) = x_2(t) \qquad 4.64$$

A simple substitution of states from the plant model would yield the following input-output model for the primary process

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t-k) + M(q^{-1})\, v(t-1) \qquad 4.65$$

where

$$A(q^{-1}) = 1 - a_1 q^{-1} \quad , \quad B(q^{-1}) = b_1 q^{-1} \quad , \quad M(q^{-1}) = \bar{a}_1$$

If process and measurement noise are included in the plant model, and if both measurements are available frequently, a simple Kalman filter can be constructed to obtain filtered estimates of the states and the outputs themselves. Subsequent innovations analysis would result in the following input-output relationship for the primary process

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t-k) + M(q^{-1})\, \hat{v}(t-1) + C(q^{-1})\, \omega_y(t) \qquad 4.66$$

On the other hand, if measurements of the primary output are available only intermittently, then the Kalman filter would have to be modified accordingly to accommodate the infrequent primary output measurements and the frequently available secondary output and control input measurements. This can be done according to the filter proposed in section 4.4. Subsequent innovations analysis would result in the following multirate formulation for the primary process

$$A_J(q^{-J})\ y(t) = B_J(q^{-1})\ u(t-k) + M_J(q^{-1})\ \hat{v}(t-1) + C_J(\bar{q}^{J})\ \omega_y(t) \qquad 4.67$$

where the polynomials, according to equation 4.47, would be given by

$$A_J(q^{-J}) = [\ 1 - (a_1)^J\ q^{-J}]$$
$$B_J(q^{-1}) = [\ 1 + a_1\ q^{-1} + ... + a_1^{(J-1)}\ q^{-(J-1)}]\ \ b_1\ q^{-1}$$
$$M_J(q^{-1}) = [\ 1 + a_1\ q^{-1} + ... + a_1^{(J-1)}\ q^{-(J-1)}]\ \ \bar{a}_1\ q^{-1}$$

It is interesting to note that, if we define a polynomial $P(q^{-1})$ as

$$P(q^{-1}) = [\ 1 + a_1\ q^{-1} + ... + a_1^{(J-1)}\ q^{-(J-1)}] \qquad 4.68$$

which is the same multirate transformation polynomial described by Lu (1989), multiplication of equation 4.66 throughout by $P(q^{-1})$ would result in the input-output relationship similar to equation 4.67, excluding noise effects.

It should be pointed out that the innovations interpretation and the multirate Kalman filter viewpoint leads to a more convenient formulation as compared to the approach of simply substituting states in the plant model followed by multiplication with the transformation polynomial adopted by Lu (1989). The convenience is because the mulirate formulation obtained in equation 4.67 describes the process in terms of available input and output measurements. More specifically, the obsever polynomial

$C_J(q^{-J})$ is conveniently expressed in terms of the infrequent sampling rate, the reason being that the associated noise term ω_y signifies noise associated with the primary output measurement which is only available infrequently.

4.7 Minimum variance multirate output estimation

The input-output expressions given in the form of equations 4.60 and 4.61 cannot be used directly for the purpose of estimation.

Consider the input-output relationship expressed by equation 4.47

$$A_J(q^{-J})\, y(t) = B_J(q^{-1})\, u(t-k) + M_J(q^{-1})\, \hat{v}(t-1) + C_J(q^{-J})\, \omega_y(t) \qquad 4.69$$

A more useful, equivalent form yielding truncation error free estimates of the primary output can be obtained by transforming equation 4.69 such that the input and output data are filtered by finite impulse response filters. In order to make the transformation, we consider the following Diophantine identity (Kucera, 1979)

$$\frac{C_J(q^{-J})}{A_J(q^{-J})} = E_J(q^{-J}) + \frac{F_J(q^{-J})}{A_J(q^{-J})}\, q^{-k} \qquad 4.70$$

$$\text{i.e} \quad E_J(q^{-J})\, A_J(q^{-J}) = C_J(q^{-J}) - F_J(q^{-J})\, q^{-k} \qquad 4.71$$

Premultiplication of equation 4.69 with $E_J(q^{-J})$ gives

$$E_j(q^{-J})A_J(q^{-J})y(t) = E_J(q^{-J})M_J(q^{-1})\ \hat{v}(t-1) + E_J(q^{-J})B_J(q^{-1})\ u(t-k)$$

$$+\ E_J(q^{-J})\ C_J(q^{-J})\ [\ \omega_y(t)\ ]$$

or

$$C_J(q^{-J})y(t) - F_J(q^{-J})y(t-k) = L_J(q^{-1})\hat{v}(t-1) + G_J(q^{-1})u(t-k)$$

$$+\ E_J(q^{-J})\ C_J(q^{-J})\ [\ \omega_y(t)\ ] \qquad 4.72$$

for $L_J(q^{-1}) = E_J(q^{-J})\ M_J(q^{-1})$

$G_J(q^{-1}) = E_J(q^{-1})\ B_J(q^{-1})$

Rearrangement of equation 4.72 yields

$$y(t) = \frac{F_J(q^{-J})y(t-k) + L_J(q^{-1})\hat{v}(t-1) + G_J(q^{-1})u(t-k)}{C_J(q^{-J})}$$

$$+\ E_J(q^{-J})\ [\ \omega_y(t)\ ] \qquad 4.73$$

In a similar manner, equation 4.61 can be transformed to yield

$$v(t) = \frac{\bar{F}(q^{-1})\ v(t-k) + \bar{G}(q^{-1})\ u(t-k) + \bar{L}\ (q^{-1})\ \hat{y}(t-1)}{\bar{C}\ (q^{-1})}$$

$$+\ \bar{E}\ (q^{-1})\ [\ \omega_v(t)\ ] \qquad 4.74$$

For optimum estimates $\hat{y}(t)$ and $\hat{v}(t)$, if the corresponding estimation errors are $e_y(t)$ and $e_v(t)$, equations 4.73 and 4.74 can be written as

$$y(t) = \hat{y}(t) + e_y(t) \qquad 4.75$$

$$v(t) = \hat{v}(t) + e_v(t) \qquad 4.76$$

$$\hat{y}(t) = \frac{F_J(q^{-J})y(t-k) + L_J(q^{-1})\hat{v}(t-1) + G_J(q^{-1})u(t-k)}{C_J(q^{-J})} \qquad 4.77$$

$$\hat{v}(t) = \frac{\bar{F}(q^{-1})\, v(t-k) + \bar{G}(q^{-1})\, u(t-k) + \bar{L}\,(q^{-1})\,\hat{y}(t-1)}{\bar{C}\,(q^{-1})} \qquad 4.78$$

$$e_y(t) = E_J(q^{-J})\,[\,\omega_y(t)\,] \qquad 4.79$$

$$e_v(t) = \bar{E}(q^{-1})\,[\,\omega_v(t)\,] \qquad 4.80$$

It should be noted that the elements of $e_y(t)$ and $e_v(t)$ are independent of all elements of the data vector. Consequently a recursive least squares algorithm can be used to obtain unbiased estimates of the parameters of equations 4.77 and 4.78 if the estimator is implemented in an adaptive framework. A simple block diagram of the adaptive inferential estimation scheme based on equations 4.75 to 4.80 is shown in Fig 4.1.

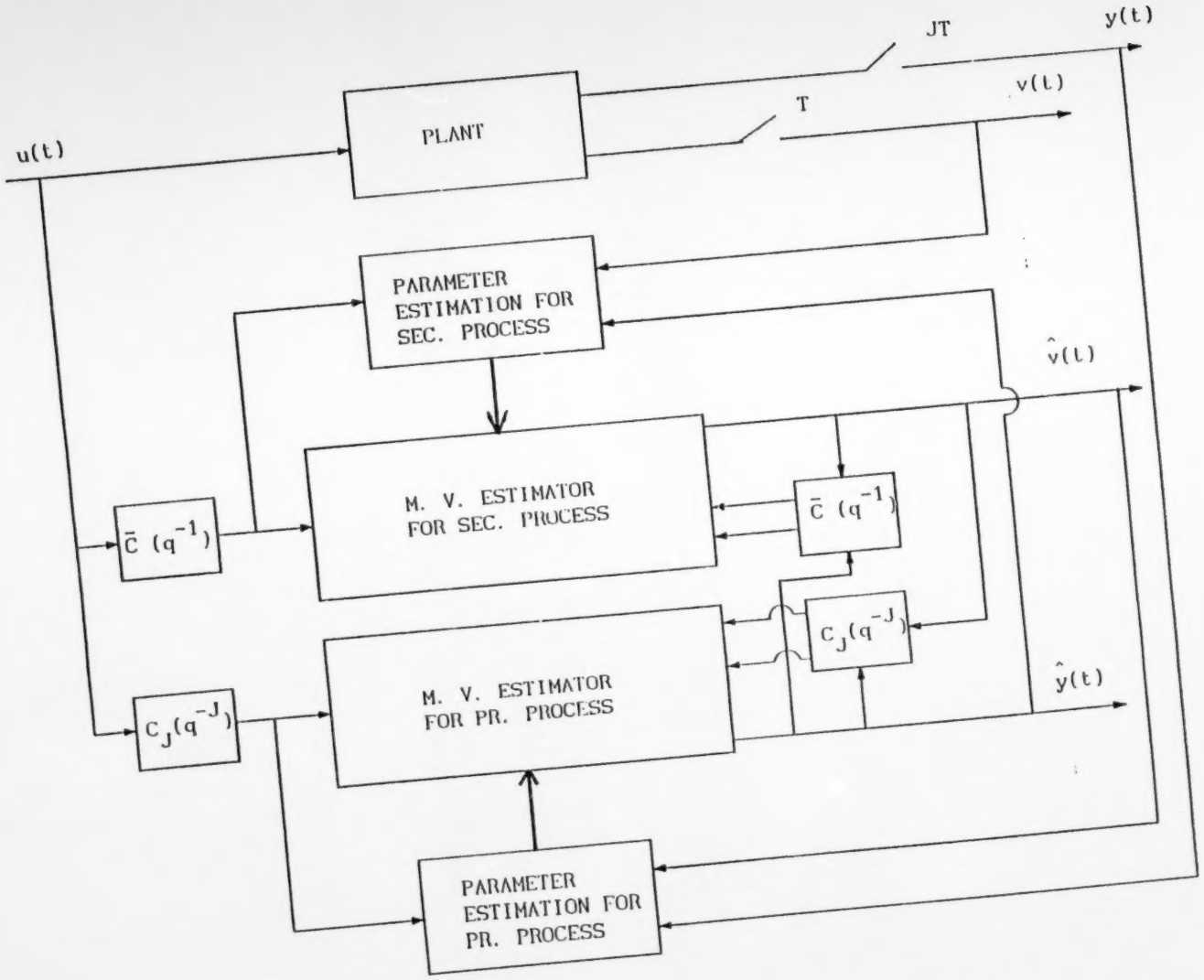

Figure 4.1 Multirate Adaptive Inferential Estimation Scheme

4.8 Multirate self tuning control

A minimum variance control law minimizes the cost function

$$J_{MV} = [y_{sp}(t+k) - y^*(t+k)]^2$$

where $y^*(t+k)$ is a k-step ahead prediction of the primary output, conditional on data up to time t. Based on the models for the primary and secondary processes given by equations 4.60 and 4.61, the k step ahead predictions for y(t) and v(t) conditional on data up to time t would be given by

$$y^*(t+k) = \frac{F_J(q^{-J})y(t) + L_J(q^{-1})v^*(t+k-1) + G_J(q^{-1})u(t)}{C_J(q^{-J})} \qquad 4.81$$

$$v^*(t+k) = \frac{\bar{F}(q^{-1})\, v(t) + \bar{G}(q^{-1})\, u(t) + \bar{L}\,(q^{-1})\, y^*(t+k-1)}{\bar{C}\,(q^{-1})} \qquad 4.82$$

Since the cost function minimizes prediction error for the primary output by manipulating a single control input, the minimum variance control law is based on a two step calculation. The steps are:

Step 1 Calculate u(t) by minimizing the cost function $[y_{sp}(t+k) - y^*(t+k)]^2$. This is achieved by setting the prediction equal to the set point in equation 4.81

Step 2 Calculate the prediction for the secondary output using the value of control input u(t) calculated in step 1.

4.8.1 Generalized minimum variance control as a multirate self tuning control law

The self tuning controller derived above is based on minimizing the output variance of the primary output. Quite often, a simple minimum variance control law ignores the practical controllability of the process and the resulting controller is sensitive to the positions of the process zeros. Clarke and Gawthrop (1975, 1979) developed a general class of control laws which broadened the class of performance objectives, retaining the implicit self tuning property.

Some versions of these generalized minimum variance controllers involve the prediction of an auxilliary output defined in terms of an user-specified transfer function $P_1(q^{-1})$. For our case, we specify $P_1(q^{-J})$ as

$$P_1(q^{-J}) = \frac{[1 - p_n q^{-J}]}{[1 - p_d q^{-J}]} = \frac{P_N(q^{-J})}{P_D(q^{-J})} \qquad 4.83$$

The coefficients p_n and p_d such that the overall gain of the transfer function is unity. For the auxilliary output defined as

$$\Psi(t) = P_1(q^{-J})\, y(t) \qquad 4.84$$

the primary process model can be modified accordingly as

$$A_J(q^{-J})P_D(q^{-J})\psi(t+k) = P_N(q^{-J})B_J(q^{-1})\ u(t) + P_N(q^{-J})M_J(q^{-1})v^*(t-1) + P_N(q^{-J})C_J(q^{-J})\omega_y(t) \qquad 4.85$$

In this case, the identity given by equation 4.70 becomes

$$\frac{C_J(q^{-J})}{A_J(q^{-J})}\ \frac{P_N(q^{-J})}{P_D(q^{-J})} = E_J(q^{-J}) + \frac{F_J(q^{-J})}{A_J(q\ )\ P_D(q^{-J})}\ q^{-k} \qquad 4.86$$

Multiplying equation 4.72 by $E_J(q^{-J})$, and repeating the analysis carried out in section 4.7, it is found that the optimal prediction of $\Psi(t+k)$ is given by

$$\Psi^*(t+k) = \frac{F_J(q^{-J})y^f(t) + L_J(q^{-1})v^*(t+k-1) + G_J(q^{-1})u(t)}{C_J(q^{-J})} \qquad 4.87$$

where $y^f(t)$ is the known signal obtained by passing y(t) through $1/\ (1 - p_d\ q^{-J})$.

The explanation for using an auxilliary output is that control of $\Psi(t)$ is easier than the control of y(t) itself. Further details on interpretation of the auxilliary output transfer function can be found in Clarke (1981).

Next, we describe a useful way of implementing the control law to prevent excessive control effort. We recall that $\Psi^*(t+k)$ is a function u(t) and can also be written

$$C_J(q^{-J})\,\Psi^*(t+k) = F_J(q^{-J})y^f(t) + L_J(q^{-1})v^*(t+k-1) + G_J(q^{-1})u(t) \quad \textbf{4.88}$$

If we can write $G_J(q^{-1})$ as

$$G_J(q^{-1}) = g_{J0} + (G_J(q^{-1}) - g_{J0})$$

where g_{J0} is the first element of the polynomial $G_J(q^{-1})$ and if we define $u^1(t)$ as the exact model following control input, then $u^1(t)$ would be calculated according to

$$y_{sp}(t+k) + [\,C_J(q^{-J}) - 1\,]\,\Psi^*(t+k) = F_J(q^{-J})y^f(t) + L_J(q^{-1})v^*(t+k-1) + [G_J(q^{-1}) - g_{J0}]\,u(t) + u^1(t) \quad \textbf{4.89}$$

But the actual prediction $\psi^*(t+k)$ is given according to

$$C_J(q^{-J})\,\Psi^*(t+k) = F_J(q^{-J})y^f(t) + L_J(q^{-1})v^*(t+k-1) + G_J(q^{-1})u(t) \quad \textbf{4.90}$$

Subtracting equation 4.90 from equation 4.89 , we have

$$y_{sp}(t+k) - \Psi^*(t+k) = g_{J0}(u^1(t) - u(t)) \quad \textbf{4.91}$$

Note that u^1 is calculated first and depending on whether there are any hard limits on u, u^1 may be clipped. In any event the u(t) actually implemented is used to update Ψ^* ready for the next sample and calculation for u^1. Thus all the equations apply no matter what the actual u(t) that is implemented. It is interesting to note that, if u(t) is implemented such that

$$u(t) = \frac{u^1}{1 + (Q/g_{J0})} \qquad 4.92$$

substitution for u^1, the actual control law being implemented becomes

$$y_{sp}(t+k) - \psi^*(t+k) - Q\,u(t) = 0 \qquad 4.93$$

which is exactly the kind of control law one would expect if one penalizes the control effort in the cost function.

4.9 Parameter Estimation

The multirate model for the primary process used for identification can be written simply as

$$\Psi(t) = \frac{F_J(q^{-J})y^f(t-k) + L_J(q^{-1})\hat{v}(t-1) + G_J(q^{-1})\,u(t-k)}{C_J(q^{-J})} \qquad 4.94$$

There are two methods of dealing with the filter polynomial $C_J(q^{-J})$. It can be explicitly identified by rewriting the regression model as

$$\Psi(t) = F_J(q^{-J})y^f(t-k) + L_J(q^{-1})\hat{v}(t-1) + G_J(q^{-1})\,u(t-k) + H_J(q^{-J})\,y_{sp}(t) \qquad 4.95$$

where $H_J(q^{-J}) = 1 - C_J(q^{-J})$

However, the identification usually tends to become ill-conditioned if the set point is not persistently exciting.

The other, simpler solution is to choose an user-specified filter $C_J(q^{-J})$ or calculate $C_J(q^{-J})$ using the optimal Kalman gains according to equation 4.47 to filter the noisy measurements. Calculation of the steady state Kalman gains themselves would require knowledge of variances of the process and measurement noises.

Parameter convergence is another serious problem to consider when we are dealing with multirate models. We note that the multirate model given by equation 4.60 is not a minimal representation of the plant. The parameter convergence problem when the identified model is not of minimal order has not been solved even for simple cases such as overparameterized DARMA models (Lu, 1989). The parameter convergence of a multirate model such as the one expressed by equation 4.60 has, however been proved by Lu (1989). No conclusion can be stated regarding covergence of the parameters to the true parameter vector. It is not difficult to see from the development of the inferential estimator model that the parameterization of the inferential equation is not unique. i.e, the identified model parameter set can belong to an equivalence class set in the parameter vector

space. However, it would be desirable that the parameters converge to an unique convergence point.

One way to resolve this problem would be to fix some parameters asssociated with each of the regressor vectors corresponding to v and u respectively.

It is important to understand that if 'self tuning' is what makes adaptive control schemes superior to fixed parameter controllers then it is precisely the same feature which can cause such algorithms to fail. There are many causes of estimator failure, and it is important that are dealt with by appropriate software, as bad parameters lead to bad control. Suggested approaches to deal with several different types of estimation failures that could possibly result while using recursive least squares based parameter estimation techniques are given by Shah and Cluett (1991).

4.10 Offset and Load disturbances

Quite often, in chemical processes one encounters disturbances of a sustained nature. These disturbances can lead to steady state offset and cause the identified model parameters to become biased. One way to deal with this is to explicitly include the estimation of d, the sustained disturbance by augmenting the data vector by 1. This implies that the regression model is of the form

$$C_J(q^{-J})\,\Psi^*(t+k) = F_J(q^{-J})y^f(t) + L_J(q^{-1})v^*(t+k-1) + G_J(q^{-1})u(t) + d \qquad 4.96$$

In most cases the offset d will be time varying, not only due to plant variations but also due to load disturbances. In this case, the following scheme is suggested by Clarke (1981) to estimate d

$$d(t) = d(t-1) + (1 - \beta^1)\, e(t-1) \qquad 4.97$$

$$\Psi^1(t) = \Psi(t) - d(t) = \underline{\Phi}(t)\,\underline{\theta} + \varepsilon(t) \text{ : regression model} \qquad 4.98$$

$$e(t) = \Psi^1(t) - \underline{\Phi}(t)\,\underline{\theta}(t) \text{ : prediction error} \qquad 4.99$$

It will be shown in the next chapter that inclusion of the bias term 'd' during identification improves the performance of the controller significantly for a nonlinear process. The possible explanation is that the extra term accounts for bias resulting from changes in the operating steady state which may be the consequence of subjecting the system to large set point changes or as a result of sustained disturbances.

Chapter 5 Multirate Inferential Estimation And Control Of A Continuous Stirred Tank Reactor

5.1 Introduction

In this chapter, simulation of the control of the reactant conversion in a continuous stirred tank reactor is studied to evaluate the estimation and control formulations developed in Chapter 4. The continuous stirred tank reactor being studied is shown in Figure 5.1. The continuous stirred tank reactor provides a good example for the study of the performance of inferential estimation and control algorithms because while the reactor temperature can be measured as frequently as desired, the reactant concentration in the reactor can be assumed to be measured only infrequently due to possible limitations of on-line composition analyzers. Mechanistic models relating the reactor temperature and reactant concentration to the control input (the coolant temperature) and to each other can be easily developed.

The multirate inferential estimation algorithm developed in Chapter 4 is used to infer the reactant concentration from the frequent measurements of the reactor temperature. The generalized minimum variance controller, adapted for multirate self tuning control, developed in Chapter 4 is used to control the reactant conversion level. The frequently available measurements of the reactor temperature and the coolant temperature along with the infrequently available measurements of the reactant concentration

are used to optimally update the parameters of the estimation and control algorithms.

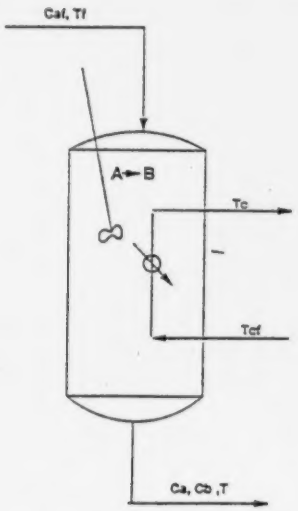

Figure 5.1 Schematic diagram of the continuous stirred tank reactor

5.2 Process Model

The dynamics of a continuous stirred tank reactor can be described using the principles of conservation of mass and energy. In the simplest case of a material component A undergoing a reaction with first order kinetics in a controlled temperature environment, the balance equations based on those given by Ray (1981) can be written as

$$V \frac{dCa}{dt} = (C_{af} - C_a) F - V k \exp(-E/R T_R) C_a \qquad 5.1$$

$$V\rho C_p \frac{dT_R}{dt} = F\rho C_p (T_f - T_R) - (\Delta H) V k_0 \exp(-E/RT_R) C_a - h A(T_R - T_c) \qquad 5.2$$

The balance equations can be expressed in a general or non-specific form by transforming all the variables involved into dimensionless quantities. Defining dimensionless variables as

$$x_1 = \frac{(C_{af} - C_a)}{C_a} \qquad x_2 = \frac{T_R - T_f}{T_f}\gamma \qquad \beta = \frac{h\,A}{F\,\rho\,C_p} \qquad \tau = t\,(F/V)$$

$$\gamma = \frac{E}{RT_R} \qquad Da = \frac{k_o \exp(-\gamma)\,V}{F} \qquad u = \frac{T_c - T_f}{T_f}\gamma \qquad H = \frac{(-\Delta H)\,C_{af}\,\gamma}{\rho\,C_p\,T_f}$$

allows equations 5.1 and 5.2 to be expressed as

$$\frac{dx1}{d\tau} = -x1 + Da(1-x1)\exp\left[\frac{x2}{(1+(x2/\gamma))}\right] \qquad 5.3$$

$$\frac{dx2}{d\tau} = -x2(1+\beta) + H\,Da(1-x1)\exp\left[\frac{x2}{(1+(x2/\gamma))}\right] + \beta\,u \qquad 5.4$$

The model can be expressed in terms of the free ($\underline{f}$), and forced ($\underline{g}$) responses of the system as

$$\frac{d\underline{x}}{d\tau} = \underline{f}(\underline{x},\tau) + \underline{g}(u,\tau) \qquad 5.5$$

$$\underline{x} = [\,x1\,,\,x2\,]^T$$

$$\underline{f} = [\,f1\,,\,f2\,]^T$$

$$\underline{g} = [\,g1\,,\,g2\,]^T$$

where

$$f1(x,\tau) = - x1 + Da(1 - x1) \exp\left[\frac{x2}{(1+(x2/\gamma))} \right] \qquad 5.6$$

$$f2(x,\tau) = - x2(1+\beta) + H\,Da\,(1 - x1) \exp\left[\frac{x2}{(1+(x2/\gamma))} \right] \qquad 5.7$$

$$g1(u,\tau) = 0 \qquad 5.8$$

$$g2(u,\tau) = \beta\, u \qquad 5.9$$

Since $\underline{g}$ is already a linear function of u, only linearization of f1 and f2 are required. In order to linearize the model about the steady state of interest, we must establish the Jacobian of $\underline{f}$ which is given by

$$\underline{\underline{J}}^{f}(\underline{x},\tau) = \begin{bmatrix} \frac{\delta \underline{f1}}{\delta x1} & \frac{\delta \underline{f1}}{\delta x2} \\ \frac{\delta \underline{f2}}{\delta x1} & \frac{\delta \underline{f2}}{\delta x2} \end{bmatrix}$$

which can be shown to be

$$\underline{\underline{J}}^{f}(\underline{x},\tau) = \begin{bmatrix} \frac{-1}{[\,1 - x1ss\,]} & \frac{x1ss}{[\,1 + x2ss\,]^2} \\ \frac{-H\,x1ss}{[\,1 - x1ss\,]} & -(1+\beta) + \left[\frac{H\,x1ss}{[1 + x2ss\,]^2} \right] \end{bmatrix} \qquad 5.10$$

After linearization about steady state, the process model can be written as

$$\frac{d\underline{x}}{d\tau} = \underline{f}(\underline{x},\tau) + \underline{\underline{J}}^{f}_{ss} (\underline{x}(\tau) - \underline{x}_{ss}) + \underline{g}(u,\tau) \qquad 5.11$$

and since at steady state, we have

$$0 = \underline{f}(\underline{x}_{ss},\tau) + \underline{g}(u_{ss},\tau) \qquad 5.12$$

subtracting equation 5.12 from equation 5.11 and writing the model in terms of deviation variables yields

$$\frac{d\tilde{\underline{x}}}{d\tau} = \underline{\underline{J}}^{f}_{ss}\, \tilde{\underline{x}}(\tau) + \underline{g}(\tilde{u},\tau) \qquad 5.13$$

All subsequent calculations are based on the linearized model given by equation 5.13. The parameter values used are (Ray, 1981)

$$\gamma = 13.4 \qquad H = 2.5 \qquad Da = 1.0 \qquad \beta = 0.5$$

The steady state values of the temperature of the feed and that of the coolant are

$$T_f = 440\ K \qquad T_c = 419\ K$$

Based on these values of steady state inputs, the steady

state values for the state are calculated by solving the equations

$$0 = -x1ss + Da(1 - x1ss)\exp\left[\frac{x2ss}{(1 + (x2ss/\gamma))}\right] \qquad 5.14$$

$$0 = -x2ss\,(1+\beta) + H\,Da(1 - x1ss)\exp\left[\frac{x2ss}{(1 + (x2ss/\gamma))}\right] + \beta\, uss \qquad 5.15$$

to obtain

$$x1ss = 0.7127 \quad ; \quad x2ss = .9747$$

These values correspond to a steady state conversion of 0.7127 and a steady state reactor temperature of 471.988 K. The steady state Jacobian based on these values, according to equation 5.10 is

$$\underline{\underline{J}}^{f}_{ss} = \begin{bmatrix} -3.4809 & 0.6194 \\ -6.2022 & 0.0484 \end{bmatrix} \qquad 5.16$$

so the linearized process model can now be written as

$$\begin{bmatrix} \dot{\tilde{x}}1 \\ \dot{\tilde{x}}2 \end{bmatrix} = \begin{bmatrix} -3.4809 & 0.6194 \\ -6.2022 & 0.0484 \end{bmatrix} \begin{bmatrix} \tilde{x}1 \\ \tilde{x}2 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} \tilde{u} \qquad 5.17$$

5.3 Stability analysis

It is well known that solutions for continuous stirred tank reactors may yield multiple steady states (Friedly, 1972). The stability of these solutions can be typically analyzed by plotting the energy generation and removal rate versus temperature. The energy generation is due to the heat generated by the reaction given by the term $-(\Delta H)\ V\ k_0 \exp(-E/RT_R)\ C_a$ in the energy balance equation 5.2 , while the energy removal is due to the sum of the cooling effects resulting from the passage of coolant fluid given by the term $h\ A(T_R - T_c)$ and from the continuous flow of the feed stream into the reactor given by the term $F\rho\ C_p\ (T_f - T_R)$ in equation 5.2. Figure 5.2 is a plot which illustrates the stability of the steady state, with the intersection of the two curves representing the steady state solution. In our case, since the slope of the heat removal curve is greater than the slope of the heat generation curve at the steady state, the given steady state is probably stable. A simple physical interpretation of the relationship between the heat generation (or removal) and the stability of the operating steady state is that if, for a slight increase in temperature, the heat generation was greater than the heat removal, the temperature would rise even higher and would not return to the same steady state. i.e the operating steady state would be considered as being unstable. An analogous argument would apply for a slight decrease in temperature. The implication is

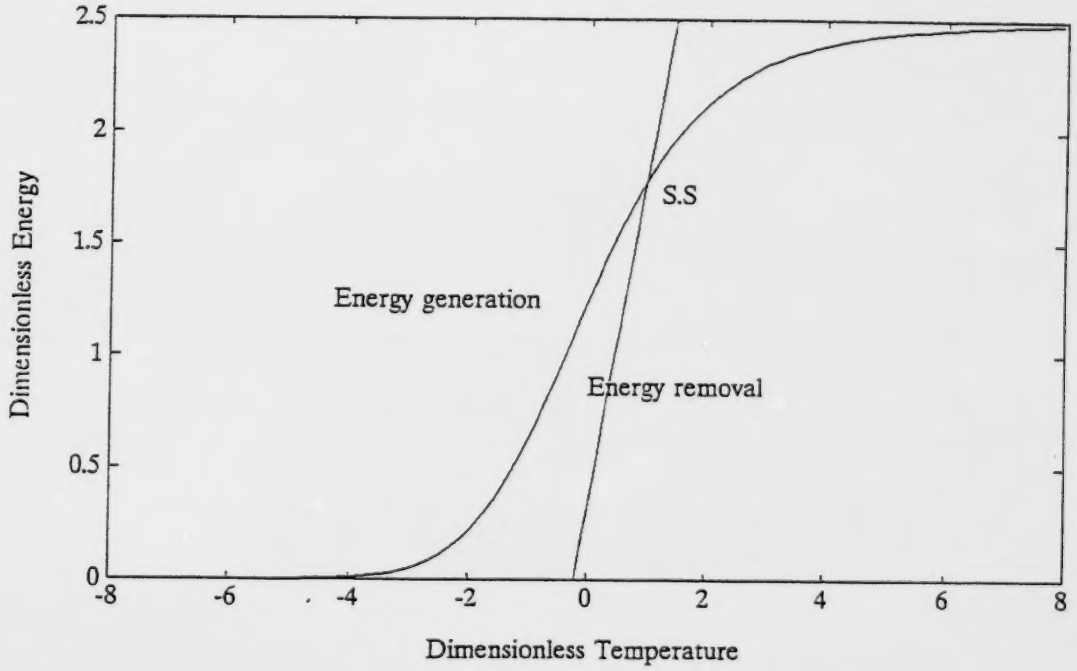

Figure 5.2 Energy generation and removal for a continuous stirred tank reactor

that a necessary condition for the stability of a given operating steady state is that the slope of the heat removal curve must be greater than the slope of the heat generation curve at that point. However, this is not the only criterion for stability. The Routh-Hurwitz criterion for stability (Ray, 1981) requires that

$$\det \left(\underline{\underline{J}}^{f}_{ss} \right) > 0$$

and from equation 5.16 since $\det \left(\underline{\underline{J}}^{f}_{ss} \right) = 3.6731 > 0$, it can be concluded that the steady state in consideration is indeed stable.

5.4 Discrete-time model

If the continuous time model given by 5.17 is discretized for a sampling time step of 0.1 time units, the following discrete time model results

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} 0.7060 & 0.0523 \\ -.6217 & 1.0049 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u(t) \qquad \textbf{5.18}$$

Note that the state associated with temperature (i.e x_2) exhibits an open loop unstable pole. This is expected because of the exponential growth term in temperature associated with the rate constant. The closed loop behavior of temperature according to the discrete linearized model is however found to be stable.

The outputs are defined as reactant coversion (y) and dimensionless reactor temperature (v) expressed as deviations from the steady state values

The deterministic process model can be rewritten as

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = \begin{bmatrix} 0.7060 & 0.0523 \\ -.6217 & 1.0049 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u(t) \qquad 5.19$$

$$y(t) = x_1(t)$$
$$v(t) = x_2(t)$$

A simple substitution of states from the discrete process model would yield the following input-output model for the primary process

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t-1) + M(q^{-1})\, v(t-1)$$

where

$$A(q^{-1}) = 1 - 0.7060\, q^{-1}\ ,\quad B(q^{-1}) = 0\ ,\quad M(q^{-1}) = 0.0523$$

If process and measurement noise is included in the plant model, then, assuming that both measurements are available frequently, a simple Kalman filter can be constructed to obtain

filtered estimates of the states and the outputs themselves. Subsequent innovations analysis would result in the following input-output relationship for the primary process

$$A(q^{-1})\, y(t) = B(q^{-1})\, u(t-1) + M(q^{-1})\, \hat{v}(t-1) + C(q^{-1})\, \omega_y(t) \qquad 5.20$$

5.5 Formulation of Multirate Model

On the other hand, if measurements of the primary output are available only intermittently, the Kalman filter would have to be modified accordingly to accommodate the infrequent primary measurements. This can be done according to the filter proposed in section 4.2. Subsequent innovations analysis would result in the following multirate formulation for the primary process

$$A_J(q^{-J})\, y(t) = B_J(q^{-1})\, u(t-k) + M_J(q^{-1})\, \hat{v}(t-1) + C_J(q^{-J})\, \omega_y(t) \qquad 5.21$$

where the polynomials would be given by (cf. equation 4.54)

$$A_J(q^{-J}) = [\, 1 - (0.7060)^J\, q^{-J}]$$
$$B_J(q^{-1}) = 0$$
$$M_J(q^{-1}) = [\, 1 + 0.7060\, q^{-1} + \ldots + (0.7060)^{(J-1)}\, q^{-(J-1)}]\, 0.0523\, q^{-1}$$

In all foregoing calculations and for the purpose of simulation it is assumed that infrequent sampling period is an integral multiple of the basic sampling time, and the multiplying

integer is equal to 5. (i.e J = 5). This results in the following multirate model

$$A_J(q^{-J})\, y(t) = B_J(q^{-1})\, u(t-1) + M_J(q^{-1})\, \hat{v}(t-1) + C_J(q^{-J})\, \omega_y(t) \qquad 5.22$$

where

$A_J(q^{-J}) = [\, 1 - .1754\, q^{-5}]$

$B_J(q^{-1}) = 0$

$M_J(q^{-1}) = [\, 0.0523\, q^{-1} + 0.0369\, q^{-2} + 0.0261\, q^{-2} + 0.0184\, q^{-3} + 0.013\, q^{-1}]$

The model for the secondary process would be obtained as

$$\bar{A}(q^{-1})\, v(t) = \bar{B}(q^{-1})\, u(t-1) + \bar{M}(q^{-1})\, \hat{y}(t-1) + \bar{C}(q^{-1})\, \omega_v(t) \qquad 5.23$$

where

$\bar{A}(q^{-1}) = [1 - 1.0049\, q^{-1}]$

$\bar{B}(q^{-1}) = [0.0501\, q^{-1}]$

$\bar{M}(q^{-1}) = [-0.6217\, q^{-1}]$

5.6 Simulation results and Discussion

Estimation and control algorithms developed in Chapter 4 are implemented using a linear and a nonlinear model of the continuous stirred tank reactor process.

The following problems were explicitly considered during the implementation of these algorithms

(i) Noisy measurements

(ii) Process nonlinearites (inherently present in the process simulator when the nonlinear process model was used to simulate the continuous continuous stirred tank reactor)

(iii) Time varying process model parameters

(iv) Load disturbance effects

A representative listing of MATLAB programs related to the implementation of the multirate inferential estimation and control algorithms to the continuous stirred tank reactor is provided in Appendix B.

5.6.1 Multirate inferential estimation

The minimum variance, multirate inferential estimator is obtained by transforming equations 5.22 and 5.23 into a form where the input and output data are filtered by finite impulse response (FIR) filters (cf. section 4.7). The resulting minimum variance estimator for the primary and secondary output can be written

simply as a product of a parameter vector ($\underline{\theta}_1$) and a filtered regressor vector ($\underline{\Phi}_1^f$)

$$\hat{y}(t) = \underline{\theta}_y(t) \cdot \underline{\Phi}_y^f(t)$$

where $\underline{\theta}_y(t) = [\, f_{J1} \;\; g_{J0} \cdots g_{J4} \;\; l_{J0} \cdots l_{J4}]$

$$\underline{\Phi}_y^f(t) = [\, \hat{y}^f(t-5)\; u^f(t-1) \,..\, u^f(t-5) \;\; \hat{v}^f(t-1) \,..\, \hat{v}^f(t-5)]^T$$

and the superscript 'f' indicates that the respective signals have been filtered by $C_J(q^{-J})$. The parameter values depend on the filtering polynomial.

Similarly, for the secondary output, the estimator can be written as

$$\hat{v}(t) = \underline{\theta}_v(t) \cdot \underline{\Phi}_v^f(t)$$

where $\underline{\theta}_v(t) = [\, \bar{f}_1 \;\; \bar{g}_0 \;\; \bar{l}_0]$

$$\underline{\Phi}_v^f(t) = [\, \hat{v}^f(t-1)\; u^f(t-1)\; \hat{y}^f(t-1)]^T$$

and the superscript 'f' indicates the signals have been filtered by $\bar{C}(q^{-1})$.

The variances or noise levels of the various process and measurement noise were kept constant during all simulation runs at the following values

$$\text{var}\{\xi_y\} = 0.001 \qquad \text{var}\{\xi_v\} = 0.001$$

$$\text{var}\{\omega_v\} = 0 \qquad \text{var}\{\omega_y\} = 0.05$$

5.6.1.1 Fixed Parameter Multirate Inferential Estimation

The simulation runs presented in this section correspond to use of a fixed parameter estimator implemented under a variety of different situations. The estimator is evaluated for the linear and the nonlinear process model. In all cases, the process is excited by a series of step changes in the control input.

Case 1.1a : Linear model

The first simulation run corresponds to the case, of the filter polynomials chosen to be unity. As can be observed from Figure 5.3, due to the choice of no-filtering, the estimates of the primary and the secondary output are as noisy as the measurements themselves.

Case 1.1b : Linear model

For the simulation results presented in Fig 5.4, $\bar{C}(q^{-1})$ is chosen as $(1 - 0.8\ q^{-1})$. The effect of filtering the noisy secondary measurement on the estimation of the primary and secondary outputs is evident. In both cases, however, it is to be

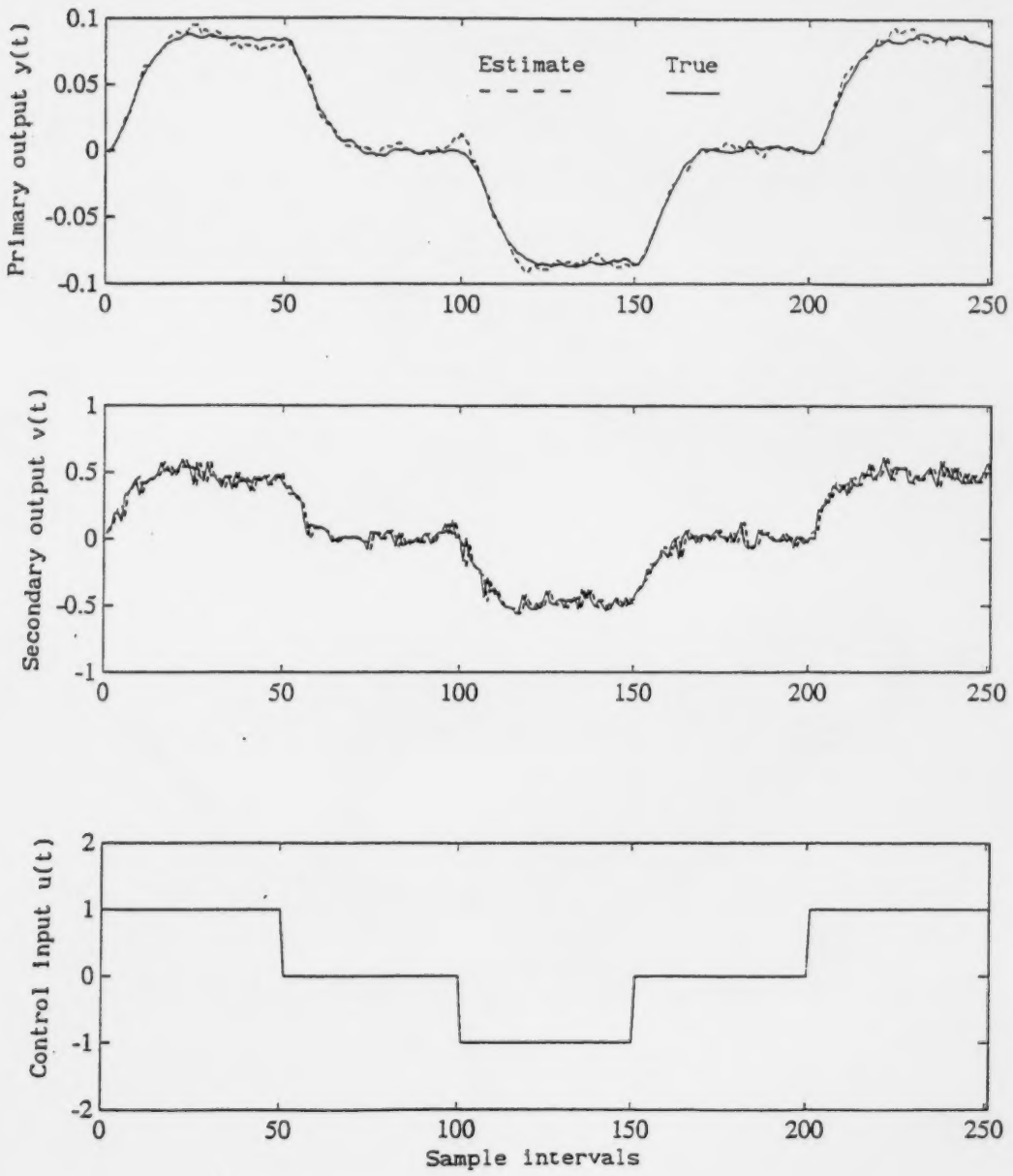

Figure 5.3 Case 1.1a: Simulation of fixed parameter multirate inferential estimation with no filtering for a linear process

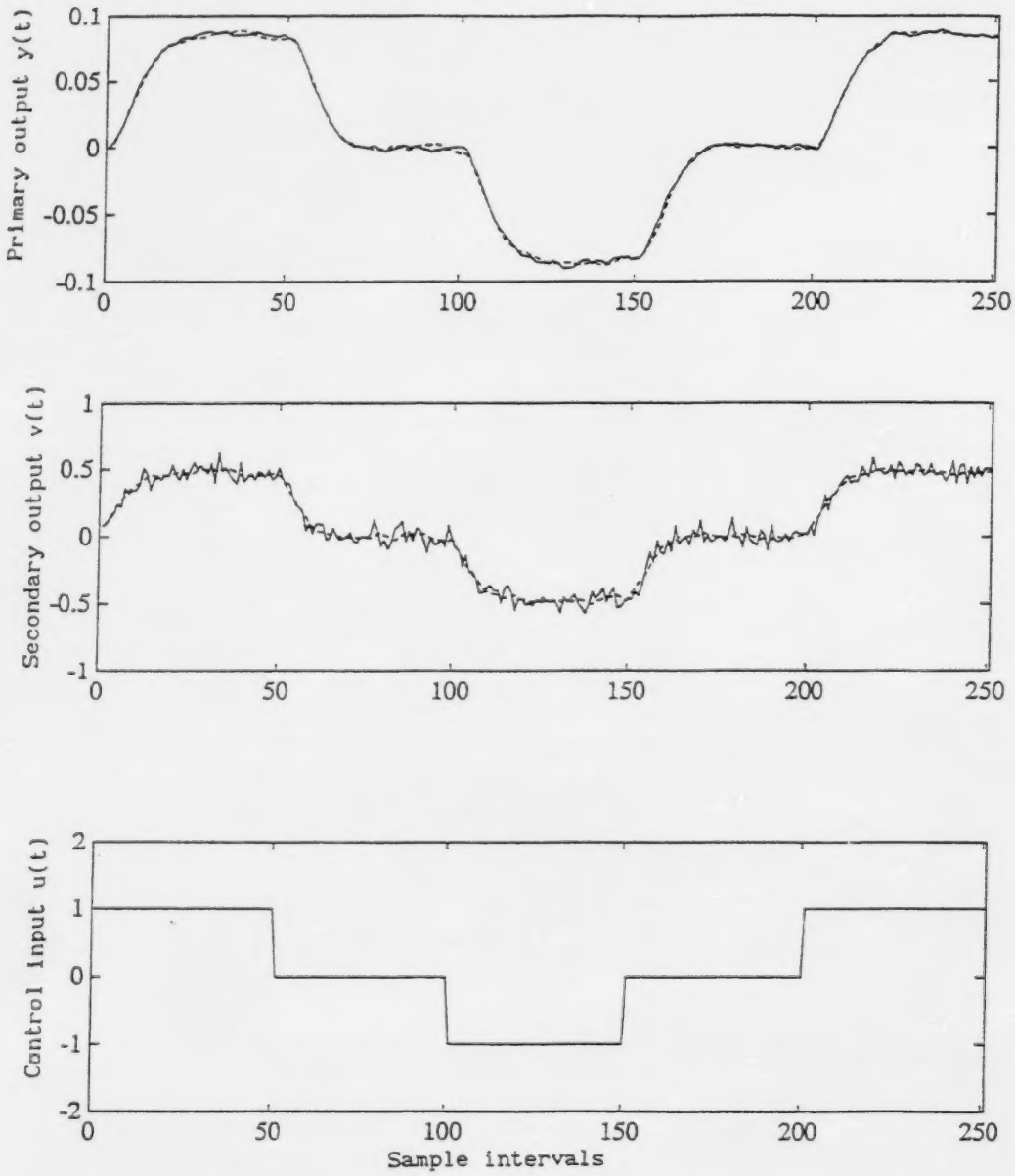

Figure 5.4 Case 1.1b: Simulation of fixed parameter multirate inferential estimation with filtering ($\bar{C}(q^{-1}) = 1-0.8q^{-1}$) for a linear process

noted that the estimates of the process outputs track the true values accurately and there is no offset.

Case 1.1c : Nonlinear model

The performance of the estimator is evaluated, using filtering, when the nonlinear process is excited by a series of step changes in the input. The step changes are large enough to ensure that the reactor is operating under conditions where the fixed parameters of the estimator, based on the linearized process model, are no longer valid. As a result it can be seen from Figure 5.5 that there is a steady state offset in the process output when the input does not return to the steady state value.

Case 1.1d : Linear model

In Figure 5.6a, the performance of the estimator is evaluated on the linear process, when the model parameters of the linear process simulator are varied with time. The parameters are varied by $\pm$ 5 % from their known values. The variation of the process model parameters is shown in Figure 5.6b. It is once again observed that under these conditions, the estimator fails to track the true process outputs.

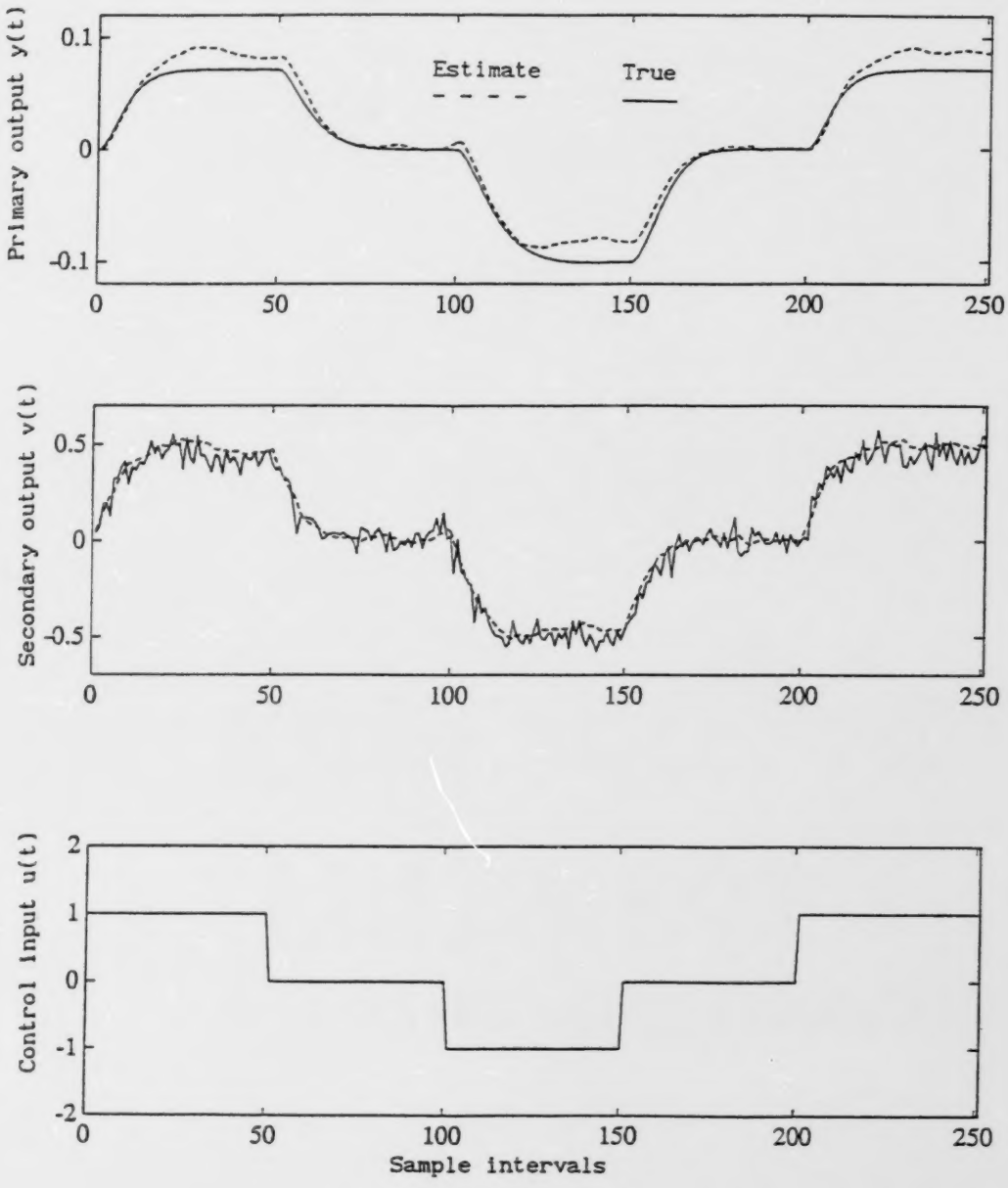

Figure 5.5 Case 1.1c: Simulation of fixed parameter multirate inferential estimation with filtering ($\bar{C}(q^{-1})= 1- 0.8q^{-1}$) for a nonlinear process

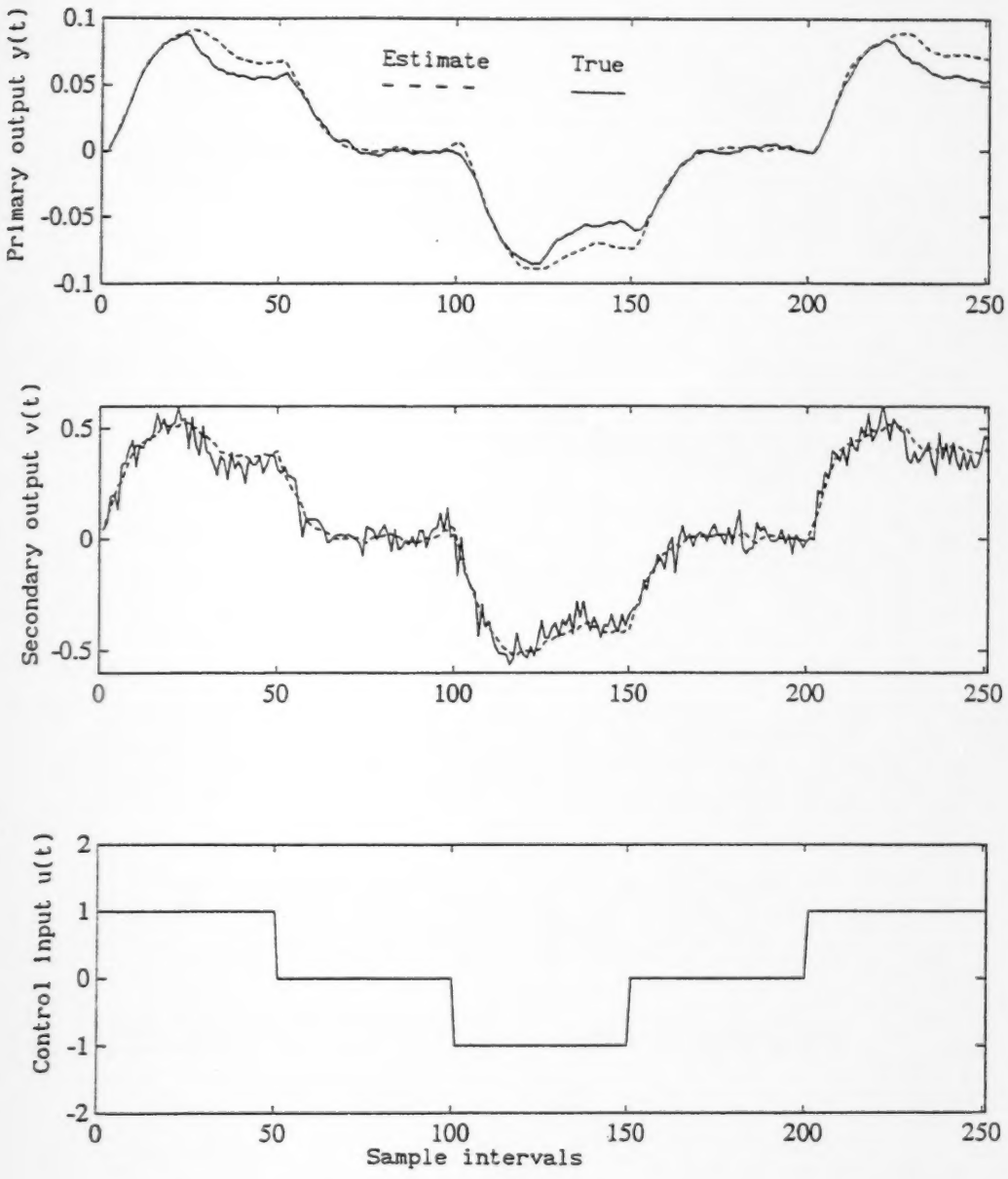

Figure 5.6a Case 1.1d : Simulation of fixed parameter multirate inferential estimation with filtering($\bar{C}(q^{-1}) = 1-0.8q^{-1}$) for a linear process with time varying parameters

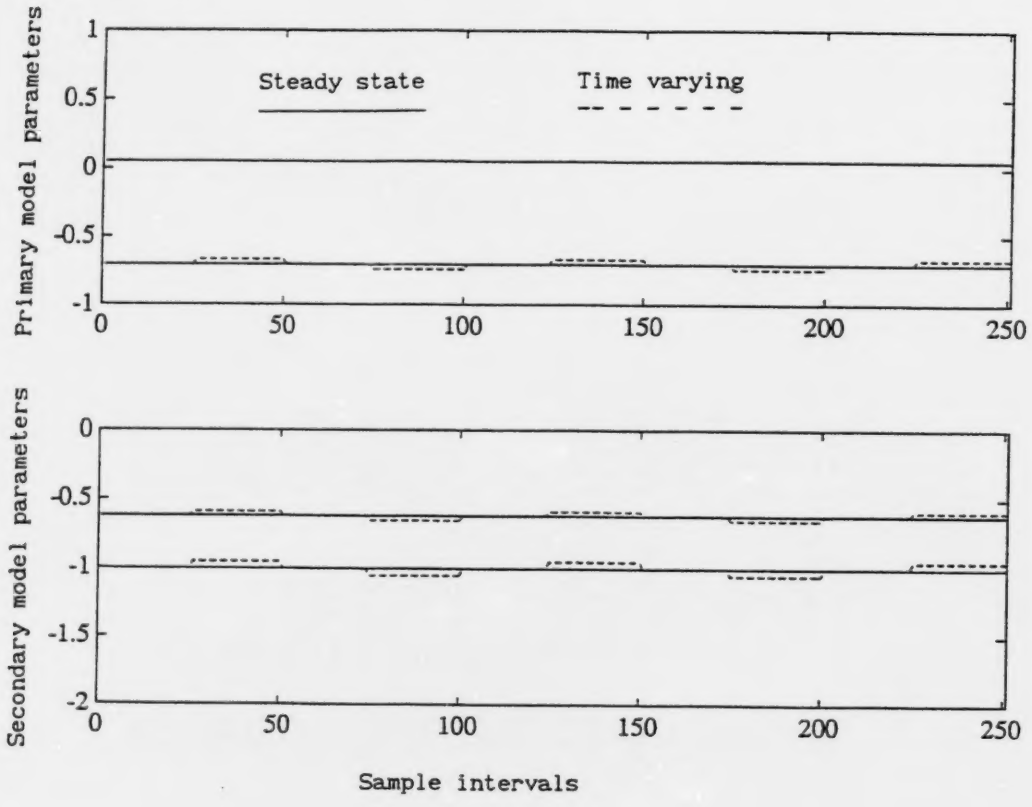

Figure 5.6b Case 1.1d : Variation of primary and secondary process model parameters with time

Case 1.1e : Linear model

In this case, the process is subjected to a time varying load disturbance input. The load disturbance transfer functions relating the effect of the disturbances on thethe primary and secondary output are arbitrarily chosen as

$$y(t) = \frac{0.02}{1 - 0.98\, q^{-1}}\, d(t) \qquad 5.24$$

$$v(t) = \frac{0.005}{1 - 0.995\, q^{-1}}\, d(t) \qquad 5.25$$

The control input is set equal to zero, so that the changes in the process output are only due to the disturbance input. The magnitude of the disturbance input is adjusted such that the minimum signal to noise ratio is as high as 20. As can readily be observed from the results in Figure 5.7, the fixed parameter estimator is unable to track the variation in the process outputs due to the time varying disturbance.

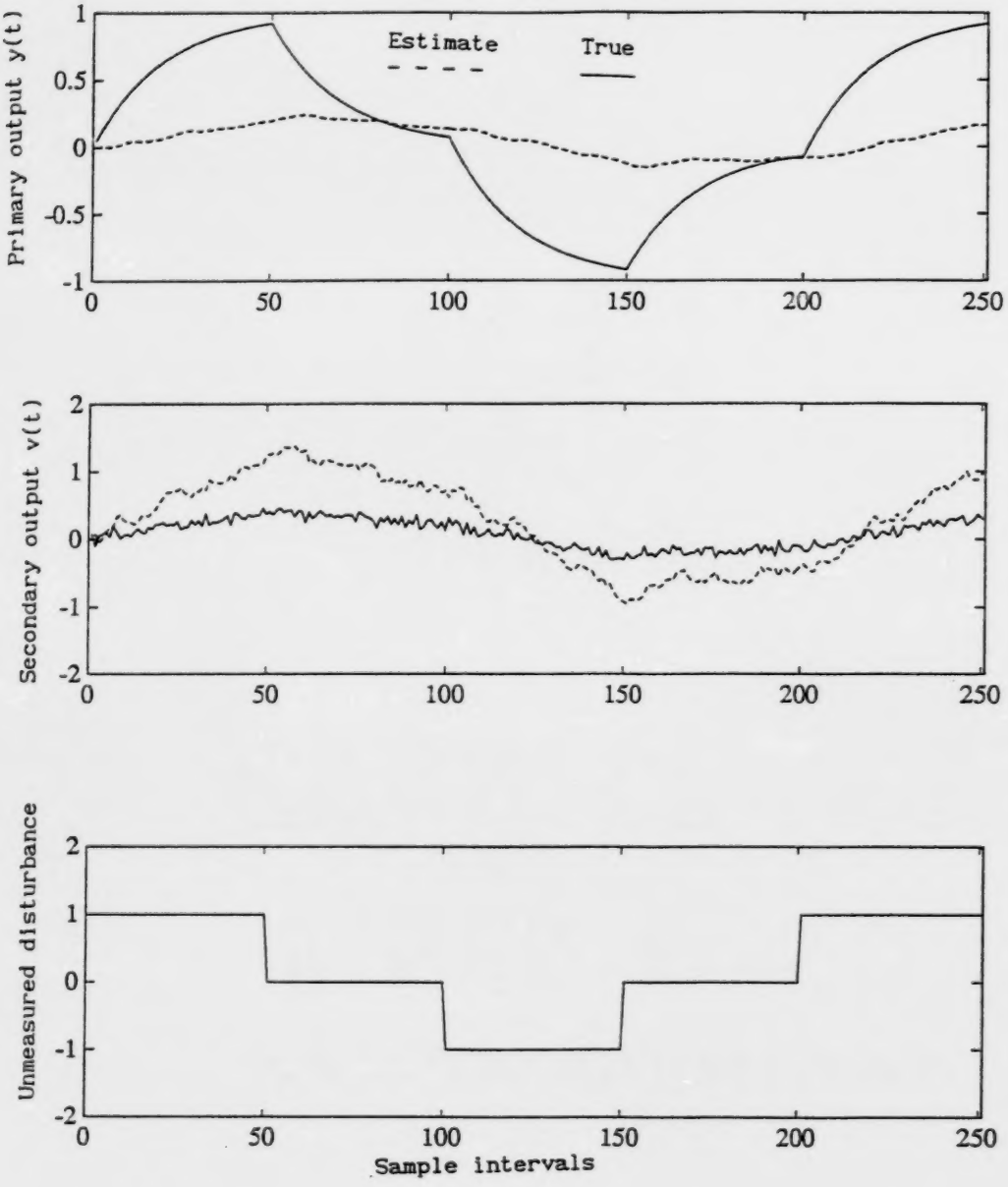

Figure 5.7 Case 1.1e : Simulation of fixed parameter multirate inferential estimation with filtering ($\bar{C}(q^{-1}) = 1-0.8q^{-1}$) for a linear process subjected to a unmeasured load disturbance

5.6.1.2 Adaptive Multirate Inferential Estimation

The simulation runs presented in this section correspond to cases when the multirate minimum variance estimator is implemented in an adaptive framework as shown in Figure 4.1 in Chapter 4. The standard recursive least squares algorithm is used to identify the parameters of the estimator models.

The standard recursive least squares algorithm is summarized in the following equations (Shah and Cluett, 1991)

Step 1 Parameter vector update

$$\hat{\underline{\theta}}(t) = \hat{\underline{\theta}}(t-1) + \underline{K}(t) [\, y(t) - \hat{\underline{\theta}}(t-1)^T \underline{\Phi}(t) \,] \qquad 5.26$$

Step 2 Gain (vector) update

$$\underline{K}(t) = \underline{\underline{P}}(t-1)\, \underline{\Phi}(t) \,/\, [\lambda + \underline{\Phi}^T(t)\, \underline{\underline{P}}(t-1)\, \underline{\Phi}(t)] \qquad 5.27$$

Step 3 Covariance matrix update

$$\underline{\underline{P}}(t) = \frac{1}{\lambda} \left[\underline{\underline{P}}(t-1) - \frac{\underline{\underline{P}}(t-1)\underline{\Phi}(t)\underline{\Phi}(t)^T\underline{\underline{P}}(t-1)}{\lambda + \underline{\Phi}(t)^T\underline{\underline{P}}(t-1)\underline{\Phi}(t)} \right] \qquad 5.28$$

Case 1.2a : Linear model

For the simulation results shown in Figures 5.8a and 5.8b, the linear process is subjected to a series of step changes in the input. The filter polynomial $\bar{C}(q^{-1})$ is chosen as $(1 - 0.8q^{-1})$. The simulation conditions are identical to Case 1.1b. In this case the parameters for the primary and secondary estimator models were intially set equal to zero. As expected, the performance of the estimator is poor during the initial adaptation period. The adaptation of the parameters themselves is shown in Figure 5.8b. The trace of the covariance update matrix in the recursive least squares algorithm is also plotted against time in Figure 5.8b. It can be seen that the magnitude of the trace converges to a zero in a very short time indicating "confidence" of the recursive least squares algorithm in the parameter estimates (Shah and Cluett, 1991).

Case 1.2b : Nonlinear model

It was seen, from Figure 5.5 (Case 1.1c) that the performance of the fixed parameter estimator deteriorated when a nonlinear process model was used to simulate the dynamics of the continuous stirred tank reactor. One of the advantages of adaptive estimation and control strategies that is always mentioned in literature (Seborg et al., 1986) is their ability to perform well in the presence of process nonlinearities. It is therefore of interest to evaluate the performance of the adaptive multirate

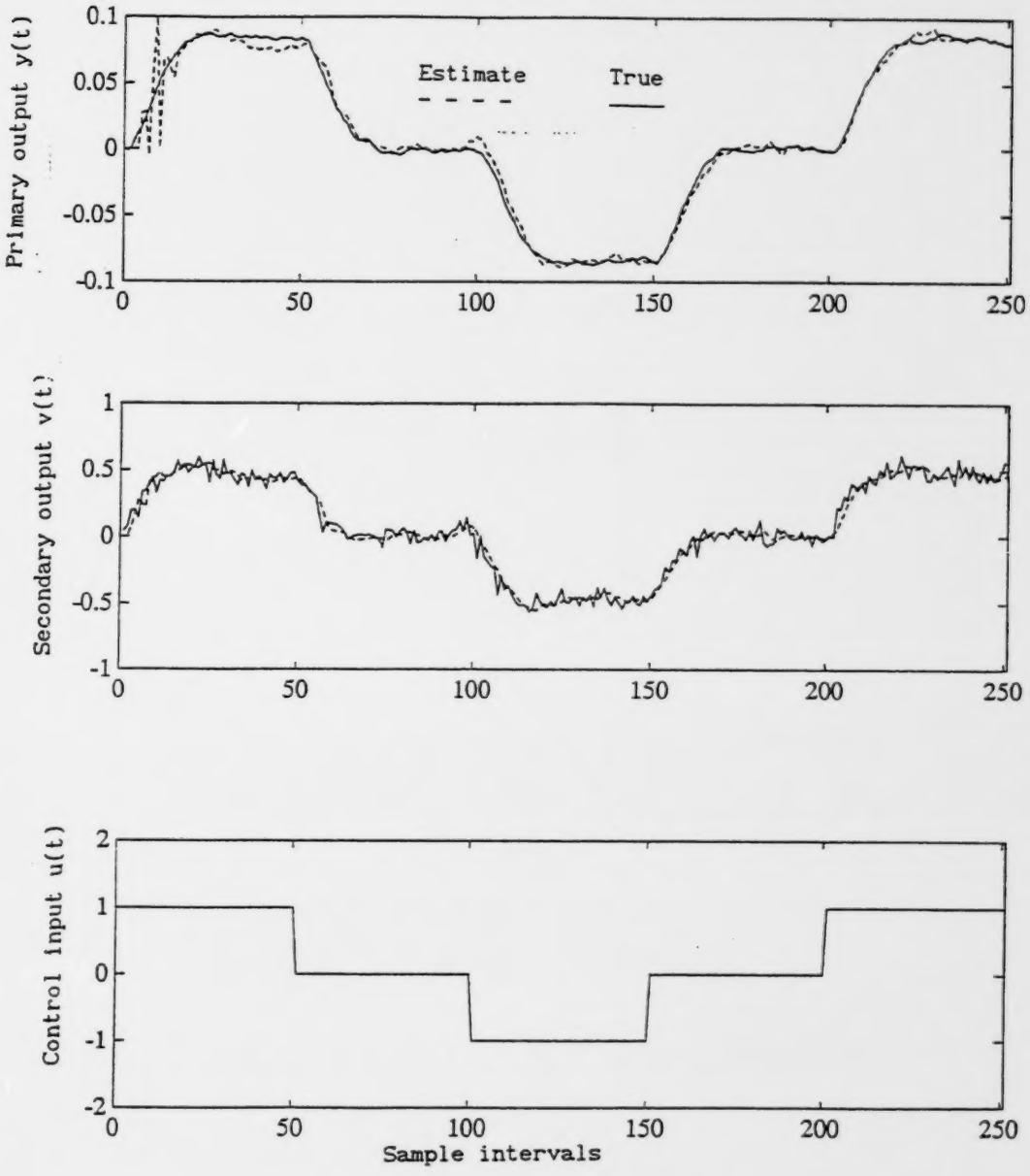

Figure 5.8a Case 1.2a : Simulation of adaptive multirate inferential estimation with filtering ($\bar{C}(q^{-1}) = 1-0.8q^{-1}$) for a linear process

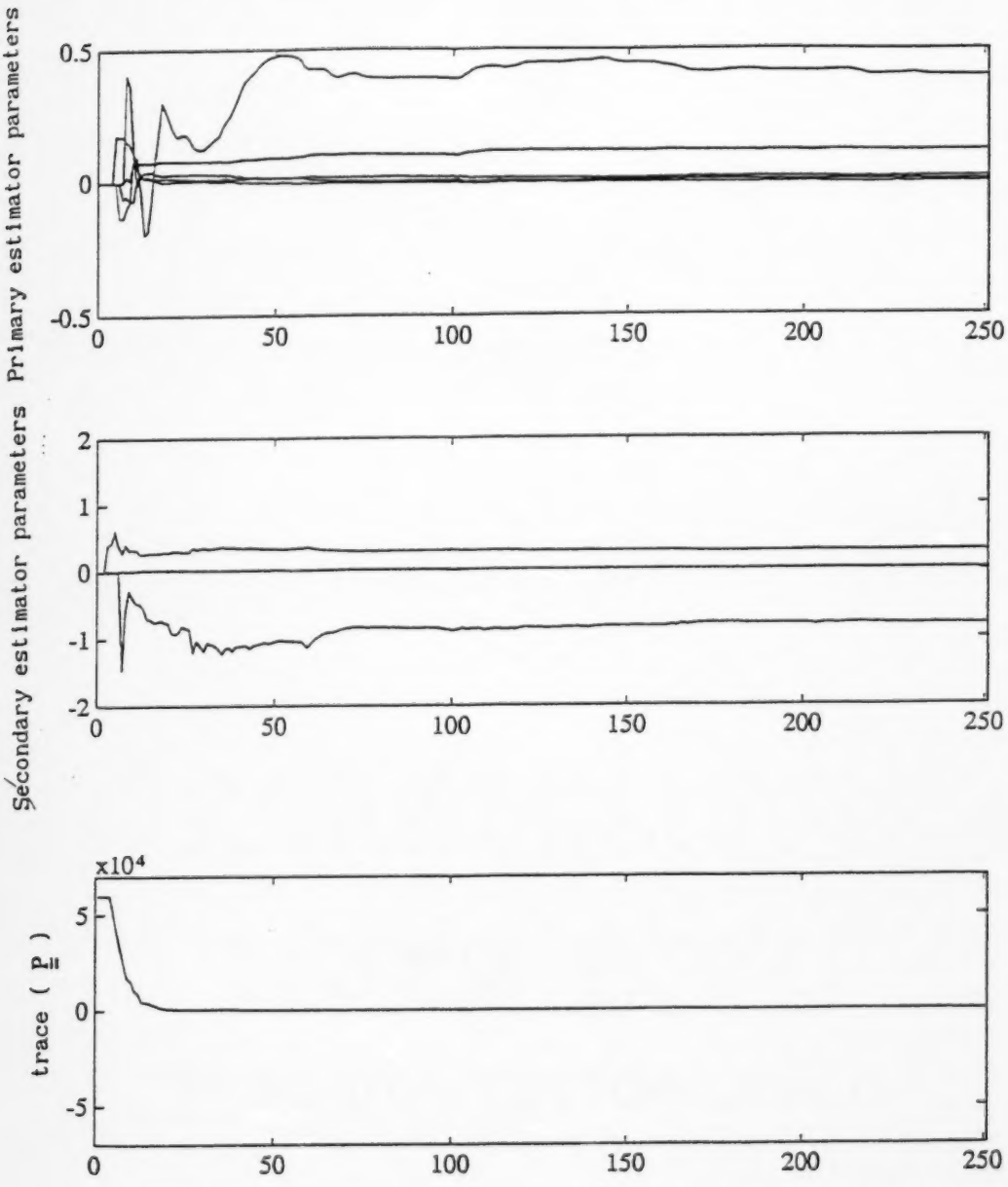

Figure 5.8b Case 1.2a : Parameter trajectories for primary and secondary estimator models models

inferential estimation strategy when a nonlinear model is used to simulate the reactor. Figure 5.9a shows the performance of the adaptive multirate inferential estimation strategy under such conditions. It can be seen that the estimator tracks the process outputs fairly accurately until $t \tilde{=} 100$ time units, after which the tracking becomes poor. Figure 5.9b shows the adaptation of the parameter estimates during the simulation run. The true values of the parameter estimates are not shown since this would require that the nonlinear process model be linearized at each sampling instant. A possible explanation for the poor performance after $t \tilde{=} 100$ time units could be that the recursive least squares algorithm fails to adapt to the changing parameters when the process outputs change direction to operating conditions lower than the initial steady state.

Numerous approaches for modifying the standard recursive least squares algorithm to improve its performance for parameter estimation have been proposed (Shah and Cluett, 1991). The two methods studied in this work are:

(i) Variable forgetting (Fortescue et al., 1981)

(ii) Covariance resetting (Goodwin et al., 1983)

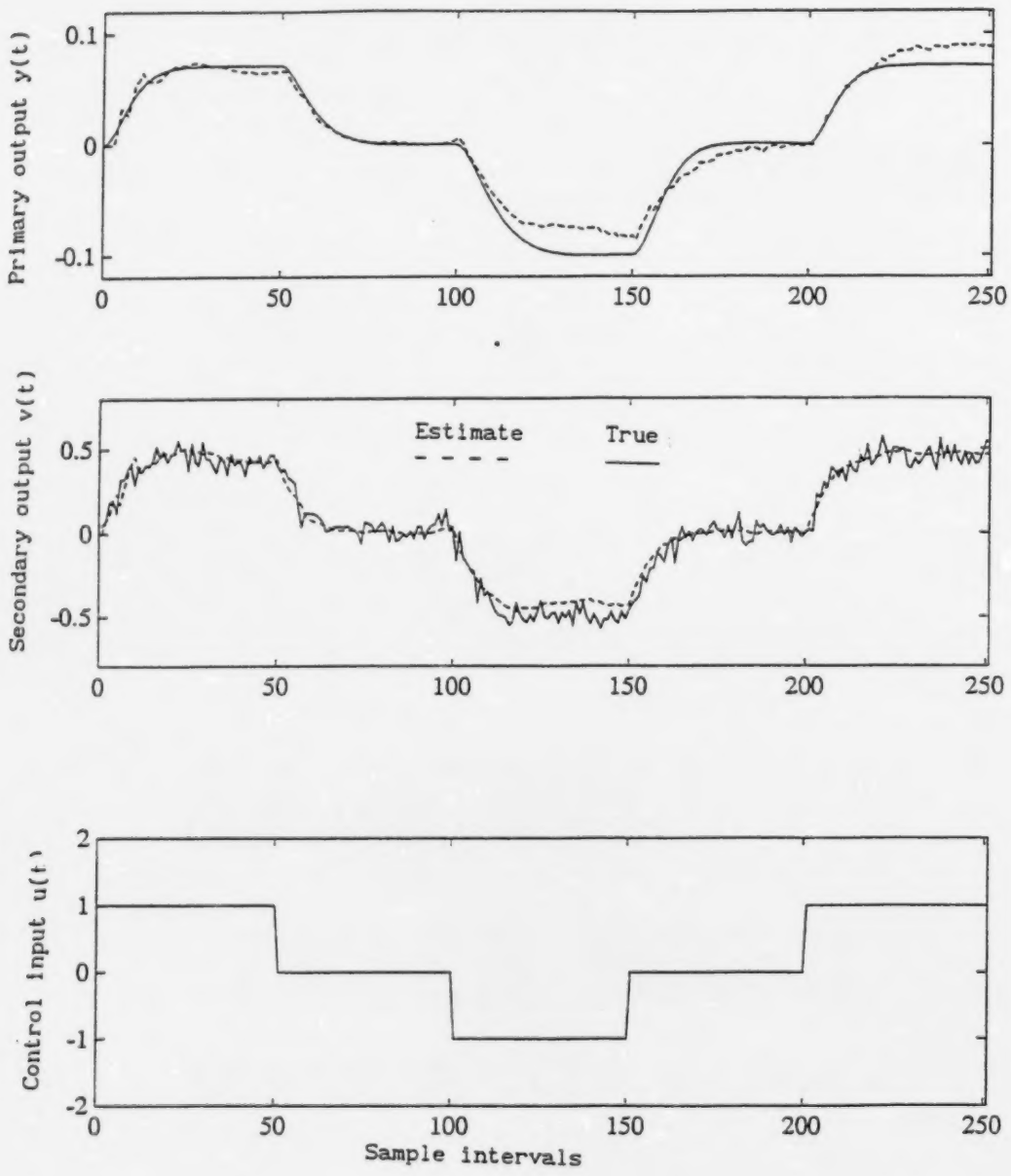

Figure 5.9a Case 1.2b : Simulation of adaptive multirate inferential estimation with filtering ($\bar{C}(q^{-1}) = 1-0.8q^{-1}$) for a nonlinear process

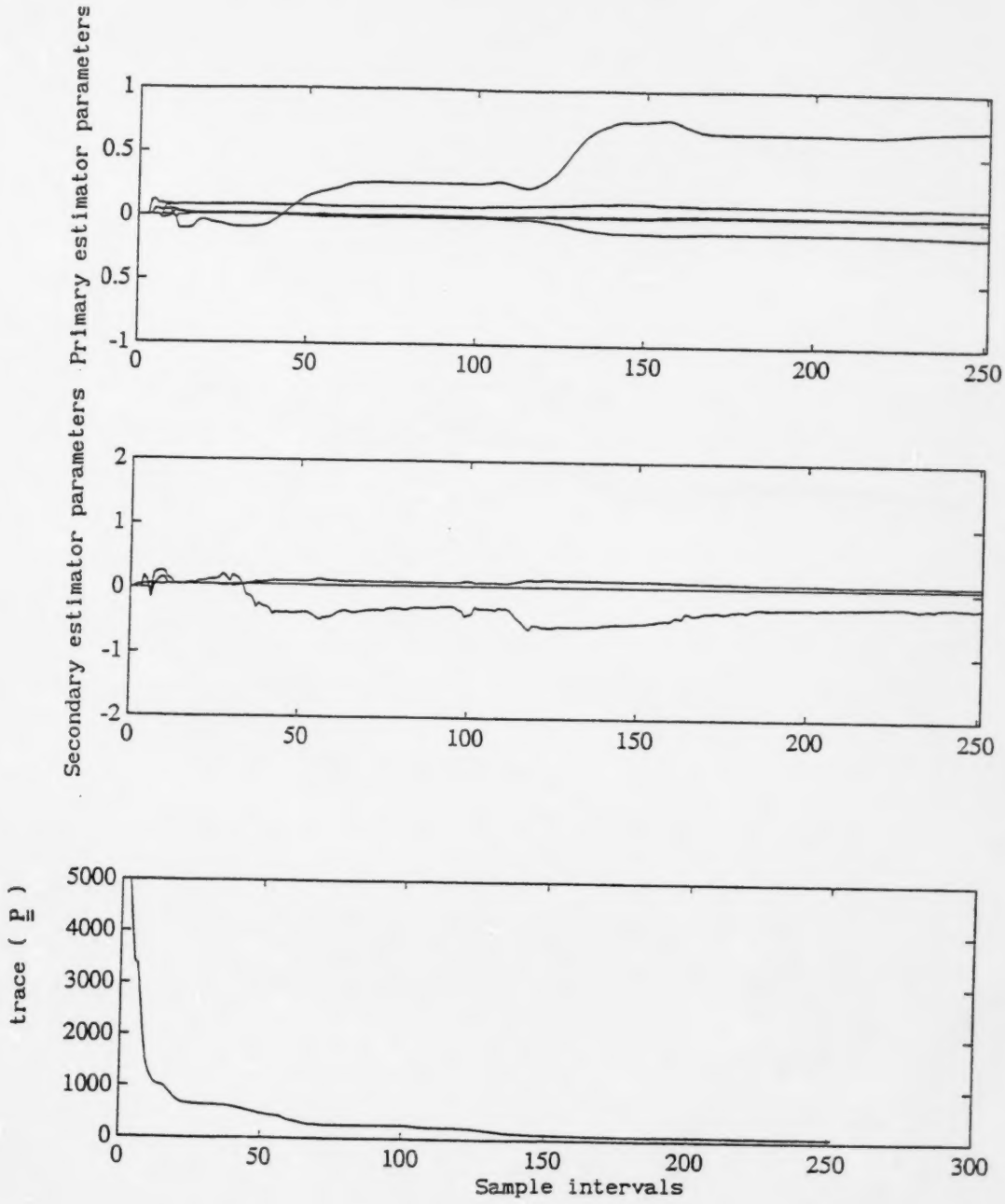

Figure 5.9b Case 1.2b : Parameter trajectories for primary and secondary estimator models

Case 1.2b.1 : Variable Forgetting :

Constant data forgetting is one method of ensuring that the identification algorithm retains its alertness. A choice of $\lambda < 1$ (cf. Eq. 5.28) results in exponential weighting, with the more recent data being given more importance or weight as compared to older data. The variable forgetting scheme differs from the the constant data forgetting strategy in that in the variable forgetting scheme, a time varying $\lambda(t)$ is automatically adjusted depending on the prediction error. So, if the prediction error is large, $\lambda(t)$ is set to a small value and $\lambda(t) \longrightarrow 1$, as the prediction error decreases. In addition, a constraint must be imposed to ensure that the covariance update matrix remains bounded, or with a variable forgetting factor, the covariance matrix can grow exponentially large (Shah and Cluett, 1991). Implementation of the variable forgetting factor algorithm (Shah and Cluett, 1991) involves replacing equations 5.27 and 5.28 by

$$\underline{K}(t) = \underline{\underline{P}}(t-1)\,\underline{\Phi}(t) / [\, 1 + \underline{\Phi}(t)^T \underline{\underline{P}}(t-1)\underline{\Phi}(t) \,] \qquad 5.29$$

$$\lambda(t) = 1 - \frac{[y(t) - \underline{\theta}^T(t-1)\,\underline{\Phi}(t)]^2}{[\, \sigma\,(\, 1 + \underline{\Phi}^T(t)\,\underline{\underline{P}}(t-1)\underline{\Phi}(t) \,]} \qquad 5.30$$

$$\underline{\underline{W}}(t) = \underline{\underline{P}}(t-1) - \underline{K}(t)\,\underline{\Phi}(t)^T \underline{\underline{P}}(t-1) \qquad 5.31$$

In order to ensure an upperbound on $\underline{\underline{P}}(t)$, the updating is

governed by

$$\underline{\underline{P}}(t) = \underline{\underline{W}}(t)/\lambda(t) \quad \text{if} \quad \text{trace of } \underline{\underline{W}}(t)/\lambda(t) < C$$

or

$$\underline{\underline{P}}(t) = \underline{\underline{W}}(t) \quad (\text{i.e. } \lambda(t) = 1)$$

for σ/σ_w selected to be a large number where σ_w is the variance of the process measurement noise.

The performance of the recursive least squares algorithm, modified using the the variable forgetting scheme with different choices for { σ , C } was studied by simulation. The simulation results for the best possible choice { σ = 10, C = 9 x 10^4 } selected by a trial and error process are presented in Figure 5.10, and although this is a slight improvement on the earlier run (Case 1.2b), the improvement is hardly significant.

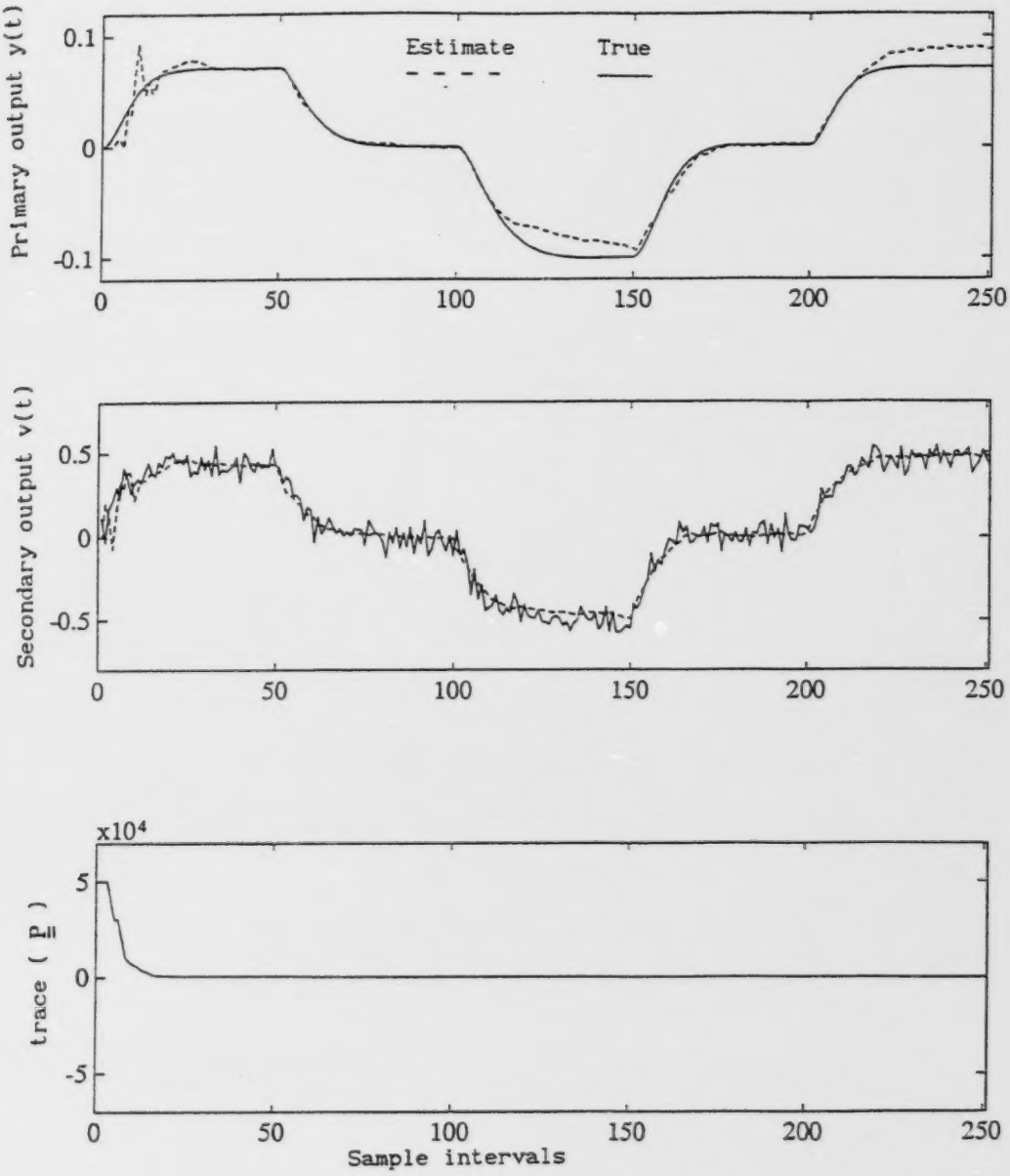

Figure 5.10 Case 1.2b.1 : Simulation of adaptive multirate inferential estimation with variable forgetting factor for a nonlinear process

Case 1.2b.2 : Covariance Resetting

In the covariance resetting scheme, the covariance update matrix is reset to a large positive non-zero value, whenever it is noticed that the algorithm has lost its drive to adapt or has 'gone to sleep'. This would occur whenever the elements of the covariance update matrix, as measured by the trace of the update matrix becomes very small. The adjustment is governed by the simple logic that if (trace ($\underline{\underline{P}}(t)$ < K_{min}) then reset $\underline{\underline{P}} = \underline{\underline{Q}}$, otherwise continue the parameter estimation using the existing covariance update matrix.

It should be noted that K_{min} is a positive number and $\underline{\underline{Q}} = q.\underline{\underline{I}}$, where q is also a positive non-zero value and $\underline{\underline{I}}$ is the identity matrix. This results in the trace of the covariance update matrix changing between between K_{min} and the trace of $\underline{\underline{Q}}$ after the initial adaptation period. The performance obtained using the covariance resetting modification with { K_{min} = 1000, q = 300 } established by trial and error is shown by simulation results presented in Figure 5.11. It is obvious that these results represent a significant improvement over those shown in Figures 5.9 and 5.10.

Comparison of the results shown in Figure 5.11 with Figure 5.6, show that the adaptive estimation scheme performs significantly better than the fixed parameter estimation scheme in the presence of process nonlinearities. Furthermore, it must be

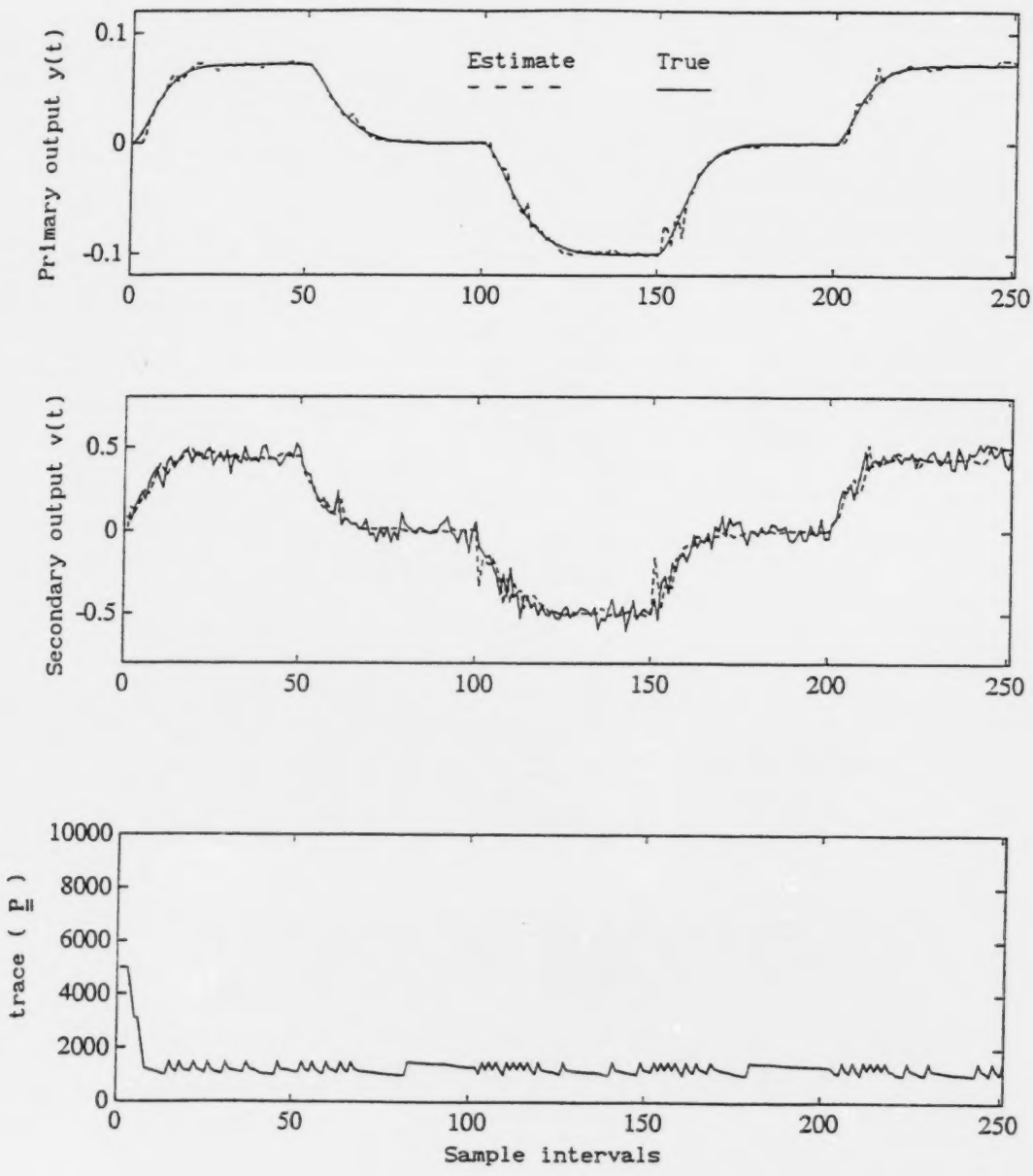

Figure 5.11 Case 1.2b.2 : Simulation of adaptive multirate inferential estimation with covariance resetting for a nonlinear process

kept in mind that the adaptive multirate inferential estimation scheme required no a-priori knowledge of the process.

Case 1.2c : Linear model

The performance of the estimator using the two different modifications to the basic algorithm was evaluated when the parameters of the linear process model are varied with time. The variation utilized was as shown earlier, for the fixed parameter case (c.f. Figure 5.6b).

Case 1.2c.1 : Variable Forgetting

Simulation results for the case when the standard recursive least squares algorithm is modified using the variable forgetting scheme are shown in Figure 5.12.

Case 1.2c.2 : Covariance Resetting

Simulation results for the case when the covariance resetting scheme is used is shown in Figure 5.13.

It can be seen by comparing the results in Figure 5.12 for the estimator using a variable forgetting factor with those in Figure 5.13, that the covariance resetting modification to the basic algorithm results in significantly better estimates of the outputs.

Comparison of the results in Figures 5.13 with those presented in Figure 5.6a (Case 1.1d) shows that the adaptive estimation scheme is easily able to cope up with time varying process parameters unlike the fixed parameter estimation scheme.

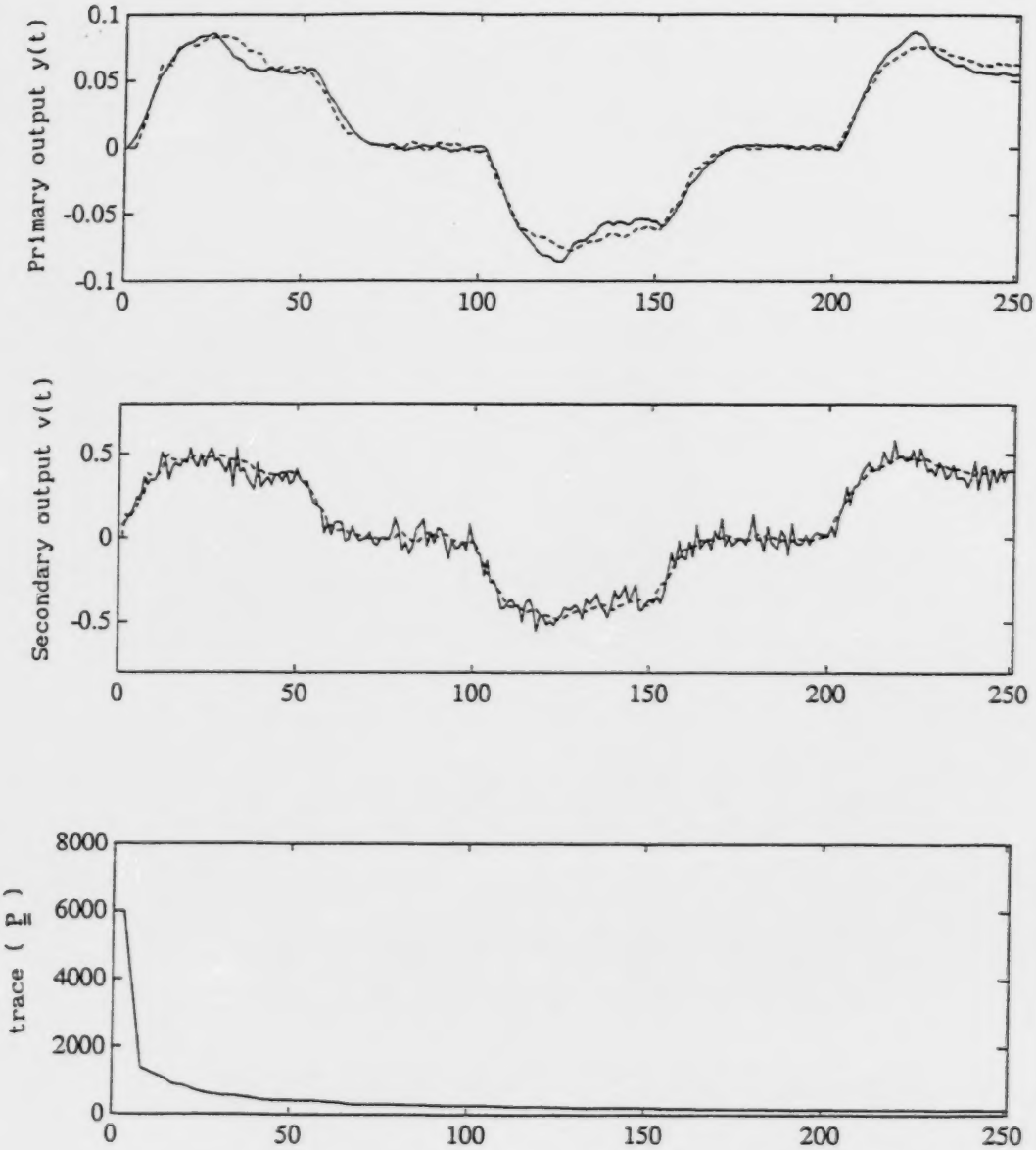

Figure 5.12 Case 1.2c.1 : Simulation of adaptive multirate inferential estimation with variable forgetting for linear process with time varying parameters

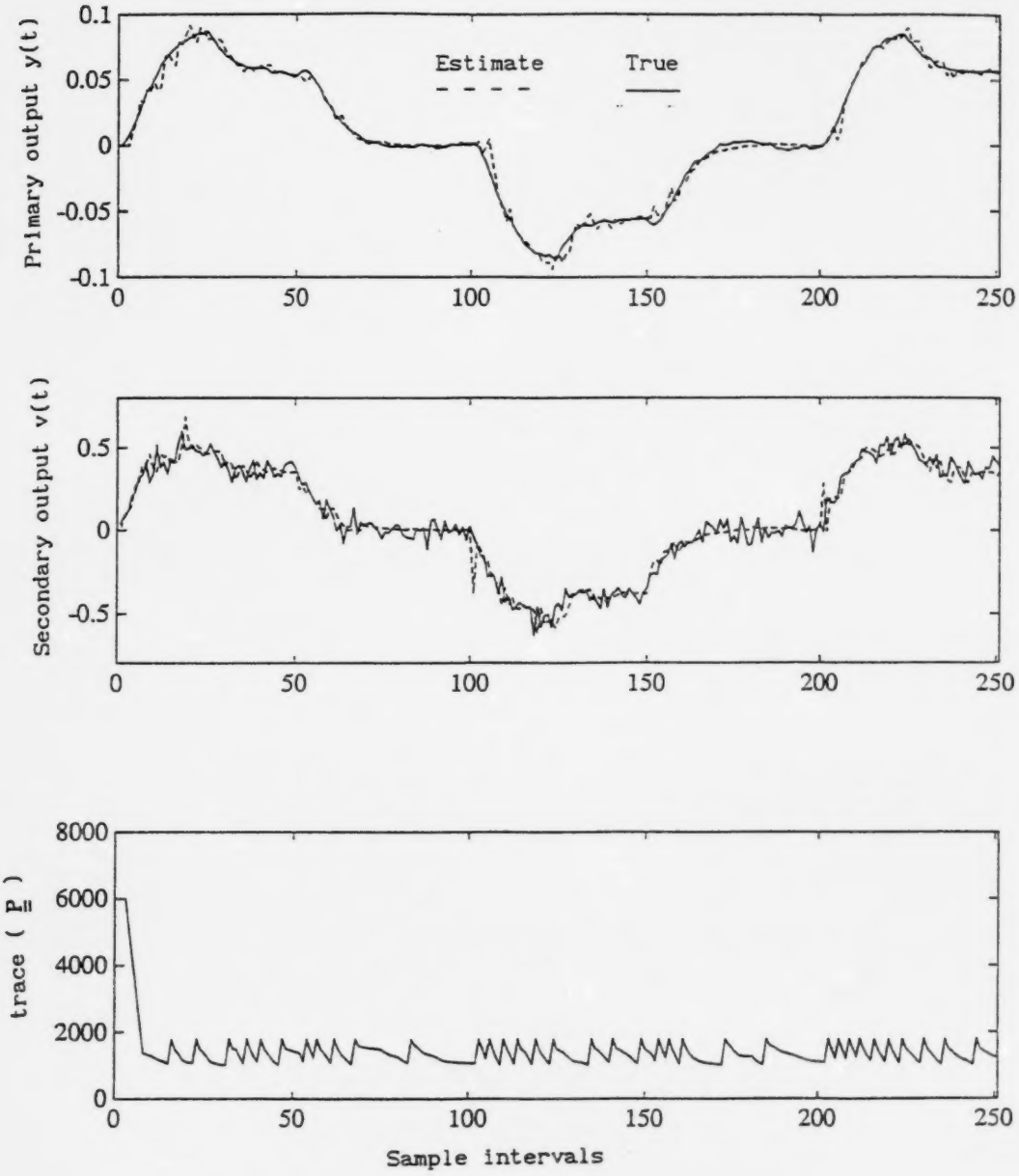

Figure 5.13 Case 1.2c.2 : Simulation of adaptive multirate inferential estimation with covariance resetting for linear process with time varying parameters

5.6.1.3 Disturbance Compensation

In an adaptive estimation scheme, the presence of a time varying load disturbance present two problems. The first problem is related to the identification of model parameters. In the presence of load disturbances, the parameter estimates tend to become biased unless the effects of these disturbances are removed. Secondly, the effects of load disturbances have to be estimated separately in order to estimate the process outputs, since in an inferential control framework, the estimates rather than the actual measurements themselves are used by the control algorithm for calculating the desired control action.

Both these problems are resolved by using the procedure suggested by Clarke (1981) that has been discussed in Chapter 4 (c.f. Section 4.10). The disturbance compensation procedure implemented in an adaptive framework is made up of three steps

Step 1 Update the disturbance estimate (or DC bias)

$$\hat{d}(t) = \hat{d}(t-1) + (1 - \beta^1)\, e(t-1) \qquad 5.32$$

Step 2 Identify the estimator models after removing the bias from the measurement

e.g., $y^1(t) = y(t) - \hat{d}(t)$

$$y^1(t) = \underline{\hat{\theta}}_y(t-1).\underline{\Phi}_y^f(t) + \varepsilon_y \qquad 5.33$$

Step 3 Estimate the prediction error

$$e(t) = y^1(t) - \hat{\underline{\theta}}_y(t).\ \underline{\Phi}_y^f(t) \qquad 5.34$$

Case 1.3a : Linear model

The performance of the multirate adaptive estimator with the disturbance compensation property incorporated for the case when the linear process is subjected to a time varying load disturbance is shown in Figure 5.14. The transfer functions relating the unmeasured disturbance input to the process outputs are the same as those utilized in Case 1.1e. Comparison of the results shown in Figure 5.14 with Figure 5.8 (Case 1.1e) shows that the compensation procedure is effective in estimating the effects of the unmeasured load disturbance. Since these estimates are used to control the variation of the primary output from the specified set point, accurate estimation would lead to better regulatory control. Furthermore, in an implicit or direct self tuning control framework, estimation of unmeasured disturbance effects is necessary to prevent parameters from becoming biased during identification.

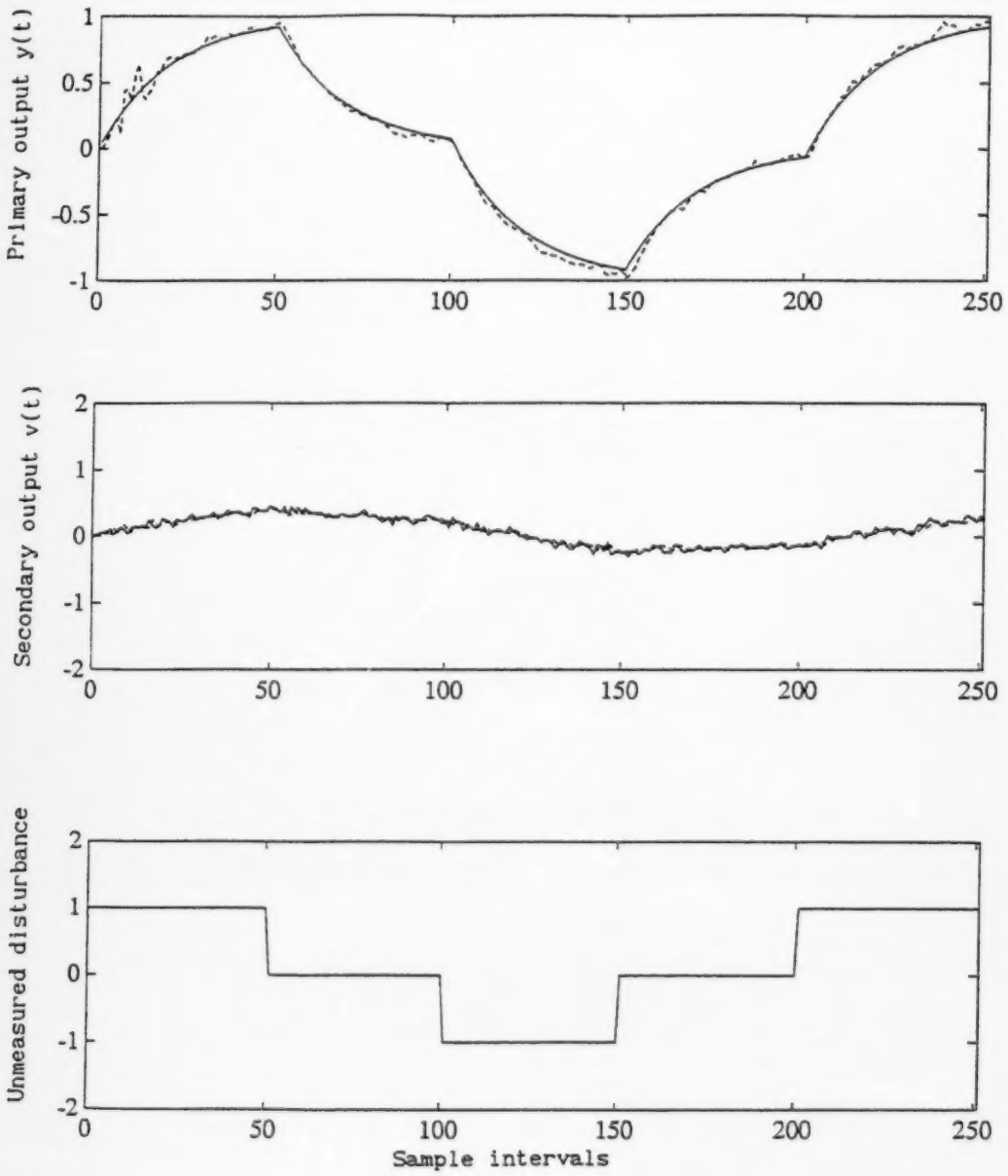

Figure 5.14 Case 1.3a : Simulation of adaptive multirate inferential estimation for linear process subjected to unmeasured load disturbance

5.6.2 Multirate inferential control

The multirate inferential generalized minimum variance control law developed in Chapter 4 involved the prediction of an auxilliary output defined in terms of a user specified transfer function $P_1(q^{-J})$. For the purpose of our simulation, for J = 5, $P_1(q^{-J})$ is specified as

$$P_1(q^{-J}) = \frac{0.1754}{1 - 0.8246\, q^{-5}}$$

with the auxilliary output defined as

$$\Psi(t) = P_1(q^{-J})\, y(t) \qquad 5.35$$

The predictions $\Psi^*(t+1)$ and $v^*(t+1)$ based on the multirate inferential generalized minimum variance control law are given by

$$\Psi^*(t+1) = \frac{F_J(q^{-J})\, \hat{y}^f(t) + L_J(q^{-1})\, v^*(t) + G_J(q^{-1})\, u(t)}{C_J(q^{-J})} \qquad 5.36$$

$$v^*(t+1) = \frac{\bar{F}(q^{-1})\, \hat{v}(t) + \bar{L}(q^{-1})\, y^*(t) + \bar{G}(q^{-1})\, u(t)}{\bar{C}(q^{-1})} \qquad 5.37$$

The multirate generalized minimum variance control law was based on a two step calculation. The first step involved the

calculation of u(t) by setting $\Psi^*(t+1)$ equal to the desired set point. The second step required that the prediction for the secondary output be calculated using the value of u(t) calculated in the first step.

In this particular case study, the primary output is not directly related to the control input. i.e $G_J(q^{-1}) = 0$. As a result, the first step in the two step control law calculation cannot be performed.

This problem was circumvented by artificially introducing the control input in the primary output prediction. This was accomplished by penalizing the control input in the control objective function. Following the approach of Clarke and Gawthrop (1975), function $\mu(t)$ was defined

$$\mu(t) \triangleq \Psi(t) + Q\, u(t-1) \qquad 5.38$$

so the control objective function then become

$$I_1 \triangleq E\{ (\Psi(t+1) + Q\, u(t) - y_{sp}(t))^2 \} \qquad 5.39$$

Obviously, I_1 is minimized by setting (Clarke, 1981)

$$(\Psi^*(t+1) + Q\, u(t) - y_{sp}(t)) = 0 \qquad 5.40$$

so, the control law sets $\mu^*(t+1)$ equal to the set point instead of $\Psi^*(t+1)$ and the equations for prediction can be rewritten as

$$\mu^*(t+1) = \frac{F_J(q^{-J})\ \hat{y}^f(t) \ + \ L_J(q^{-1})\ v^*(t)}{C_J\ (q^{-J})} \ + \ Q\ u(t) \qquad 5.41$$

and

$$v^*(t+1) \ = \ \frac{\bar{F}\ (q^{-1})\ \hat{v}(t) \ + \ \bar{L}(q^{-1})\ y^*(t) \ + \bar{G}(q^{-1})\ u(t)}{\bar{C}(q^{-1})} \qquad 5.42$$

It must be noted that in this case the two step control calculation can be implemented. However, since the control input is being penalized in the cost function, there would be a net steady state offset associated with the tracking problem. The problem of steady state offset is resolved in the following manner

The open loop steady state gain relating the primary output and the control input can be calculated from the linear deterministic model of the process (cf. equation 5.19) by eliminating the secondary output from the two equations obtained at steady state. The open loop steady state gain was found, for the given operating steady state, to be K_{ss}= 0.84153.

For the control law

$$(\Psi^*(t+1) \ + \ Q\ u(t) \ - \ y_{sp}(t)) \ = 0$$

at steady state, the offset would be given by

$$y_{sp} - \Psi_{ss} = \text{OFFSET} = Q\, u_{ss} \qquad 5.43$$

$$\text{or} \quad y_{sp} - \Psi_{ss} = \frac{Q}{0.84153}\, y_{ss} \qquad 5.44$$

Since, the primary output y and the auxilliary output Ψ are related by a steady state gain of unity, it follows that

$$y_{ss} = \frac{0.84153}{0.84153 + Q}\, y_{sp} \qquad 5.45$$

The steady state offset can be eliminated by scaling the specified set point appropriately before performing the control law calculation. This can be done in this case by scaling the given set point by the expression

$$y_{sp}^{sc} = [\, 1 + (\, Q \,/\, 0.84153) \,]\; y_{sp} \qquad 5.46$$

5.6.2.1 Fixed parameter multirate inferential control

The simulation results presented in this section correspond to cases when the multirate generalized minimum variance controller discussed earlier is implemented in a fixed parameter framework.

Case 2.1a : Linear model

The set point tracking ability of the multirate generalized minimum varaiance control algorithm using fixed parameters for estimation and control is demonstrated by the simulation results presented in Figure 5.15. The linearized process model is used to simulate the dynamics of the reactor. As shown in Figure 5.15, use of different control input penalty weights (Q) in the objective function results in steady state offset in both cases. Scaling of the set point discussed earlier has not been incorporated in the control algorithm. It can be observed that decreasing the value of Q results in

i. Making the control system more responsive

ii. Decreasing the steady state offset

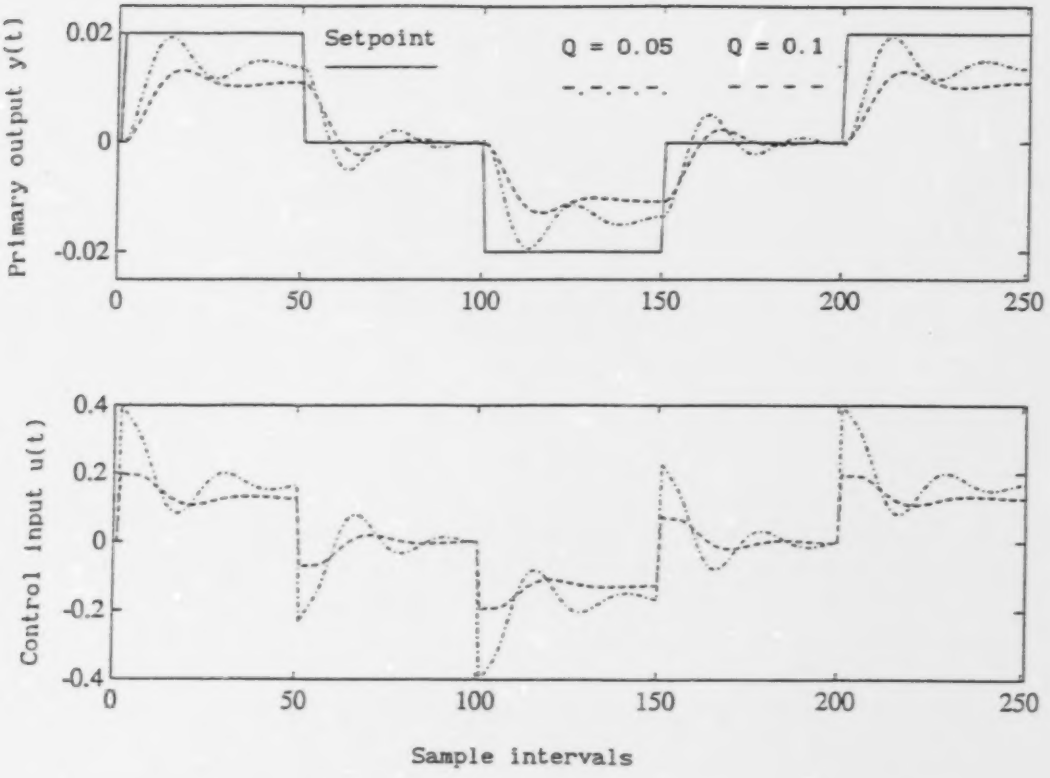

Figure 5.15 Case 2.1a : Servo tracking properties of fixed parameter inferential controllers with different penalty weights (Q) implemented on a linear process

Case 2.1b : Linear model

The effect of incorporating the set point scaling routine in the control algorithm is shown by the simulation results in Figure 5.16. As is obvious, the steady state offset is completely eliminated due to scaling.

Case 2.1c : Nonlinear model

The performance of the multirate inferential generalized minimum variance control algorithm with set point scaling using fixed parameters for estimation and control has been investigated. The resulting performance using the nonlinear model for the continuous stirred tank reactor is not satisfactory as can be observed from the simulation results presented in Figure 5.17. From the figure, it can be seen that the controller is unable to track the changes in the set point. This is because, the fixed parameter controller is unable to adapt to the changes in the operating steady state.

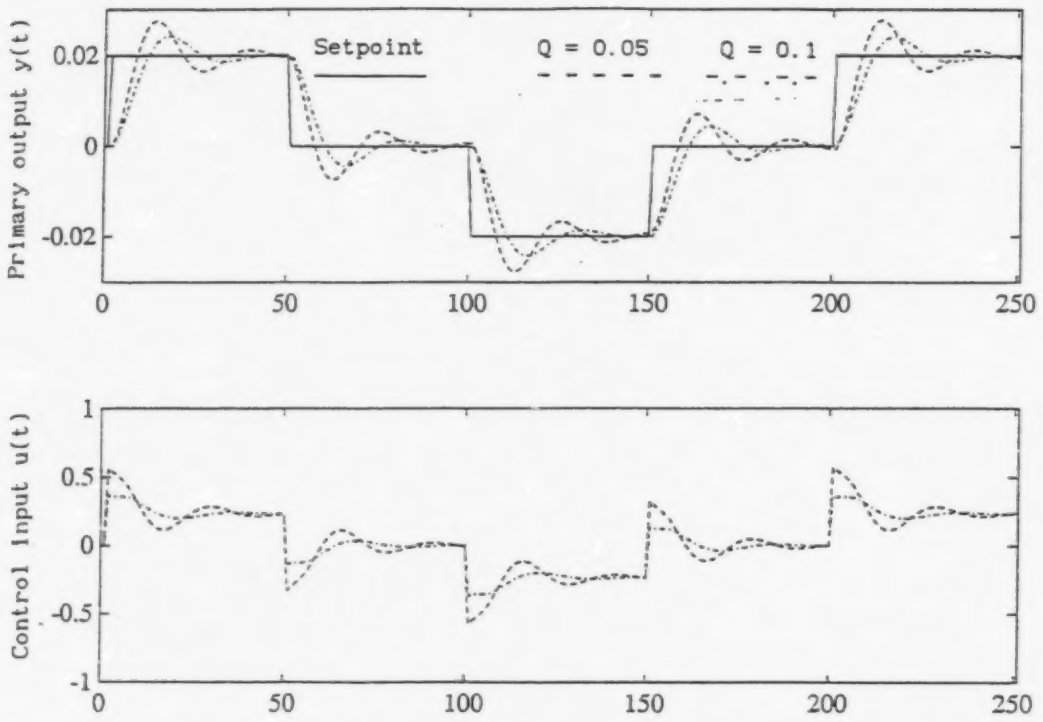

Figure 5.16 Case 2.1b : Servo tracking properties of fixed parameter inferential controller with different penalty weights (Q) with setpoint scaling implemented on a linear process

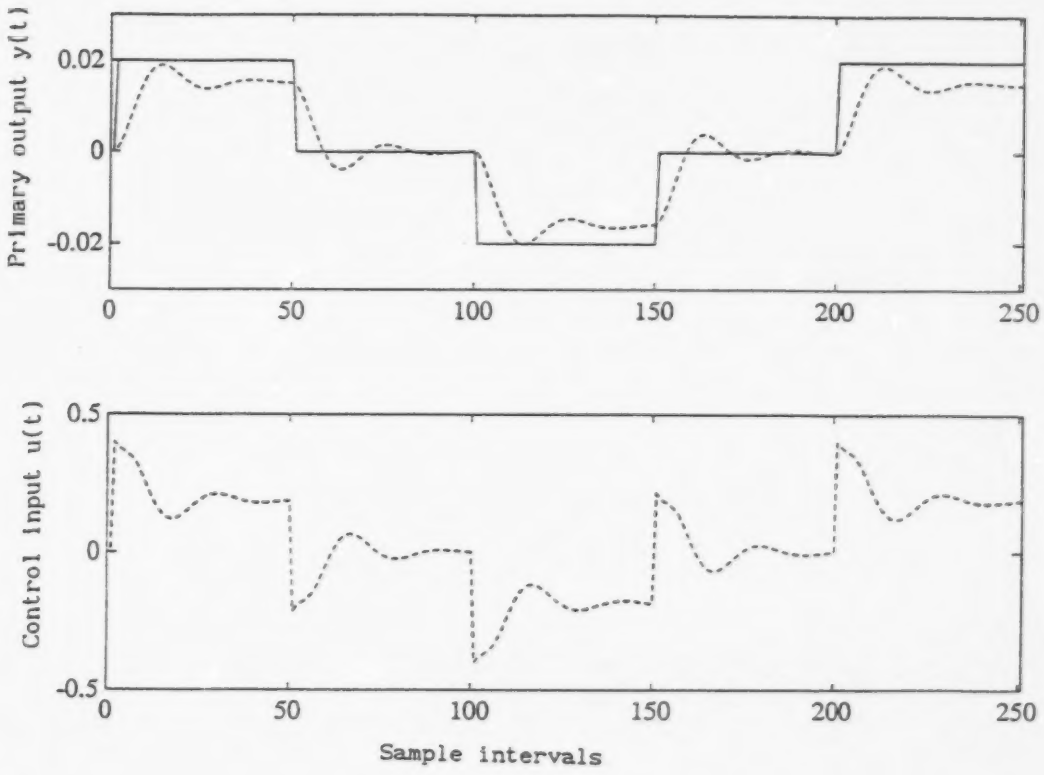

Figure 5.17 Case 2.1c : Servo tracking properties of fixed parameter inferential controller with set point scaling implemented on a nonlinear process

5.6.2.2 Adaptive multirate inferential control

When the multirate inferential generalized minimum variance controller is implemented in an adaptive framework, parameters of the multirate estimator models as well as those of the prediction models have to be identified. The model parameters for the estimator model of the primary process are different from those of the prediction models (cf. equations 4.77, 4.87) because in the latter case, the auxilliary primary output $\Psi(t)$ is used for prediction instead of the primary output y(t). Since it would be desirable to identify a single model rather than two models for the primary process, the inferential estimator is modified such that the auxilliary primary output is estimated instead of the primary output itself. The estimates of the primary output are however required during both estimation and prediction. The estimate of the primary output can be obtained from the estimate of the auxilliary primary output using the transformation

$$\hat{y}(t) = \frac{1}{P_1(q^{-J})} \hat{\Psi}(t) \qquad 5.47$$

Case 2.2a : Linear model

The simulation results when the adaptive multirate inferential control system is subjected to set point changes are shown in Figure 5.18. The setpoint scaling procedure is incorporated in the control strategy. The oscillation in the

parameters during the initial adaptation period was reduced by decreasing the magnitude of the initial covariance update matrix. The covariance resetting mechanism was also incorporated to improve adaptation. As a result, it can be seen that the adaptive multirate inferential controller with no a-priori knowledge of the process (the parameters of the estimator and prediction models were set equal to zero) does as well as the fixed prameter multirate inferential controller with known parameters (c.f. Figure 5.16).

Case 2.2b : Nonlinear model

In this case, the process is simulated a nonlinear model. The adaptive multirate inferential controller is subjected to changes in setpoint and the performance is shown in Figure 5.19. Comparison of Figure 5.19 with Figure 5.17 which corresponds to the fixed parameter case (Case 2.1c) shows that the net steady state offset due to change in the operating steady state is much smaller for the adaptive case. The steady state offset still exists, despite accurate estimation by the adaptive estimator because the set point scaling factor is no longer the same when the operating steady state is changed. It is noted (as an afterthought!) that the problem of steady state offset could have been altogether prevented if the incremental control input (Δu), instead of the control input (u) was penalized in the control objective function (Foley, 1988).

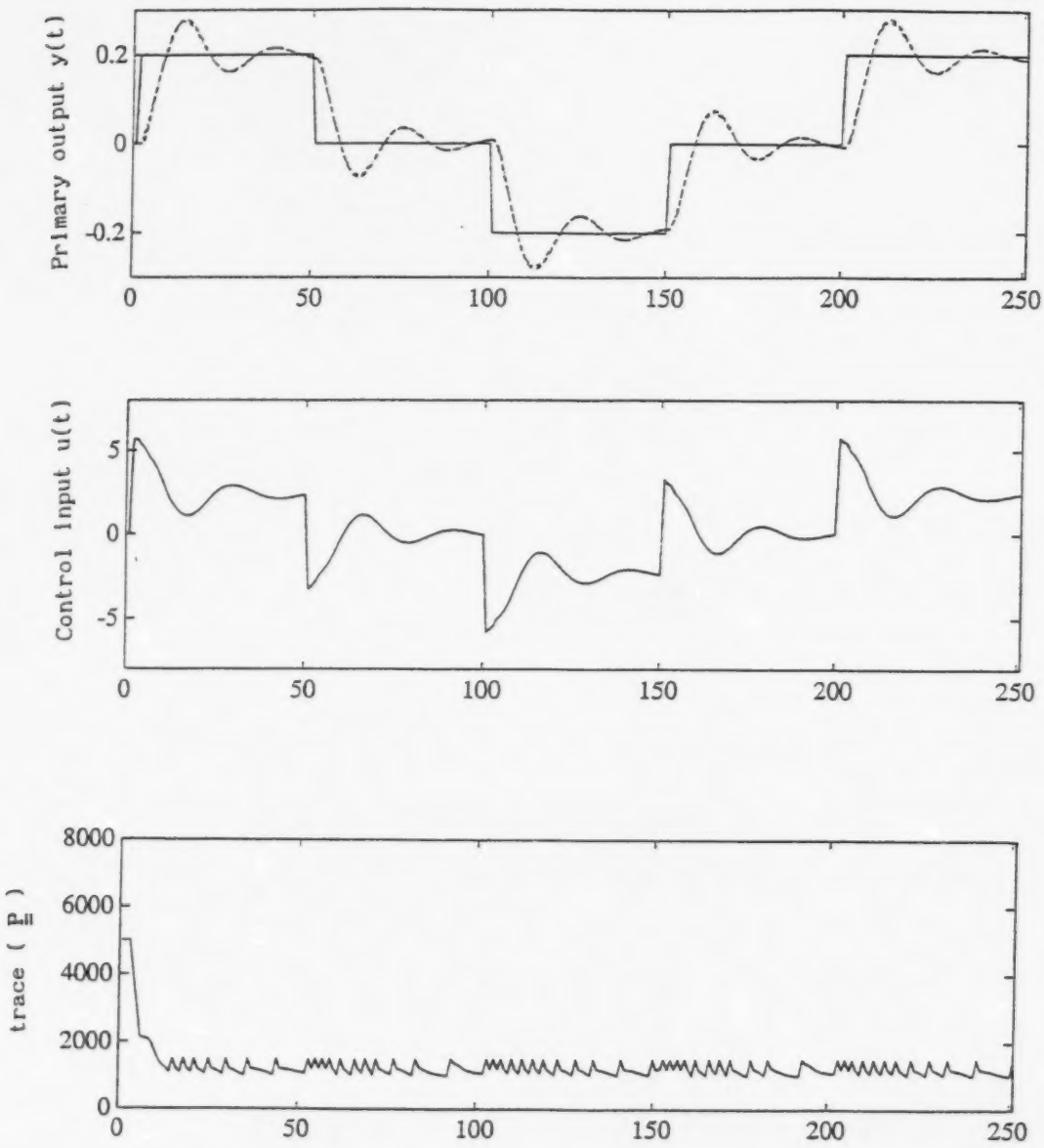

Figure 5.18 Case 2.2a : Servo tracking properties of adaptive multirate inferential controller implemented on a linear process

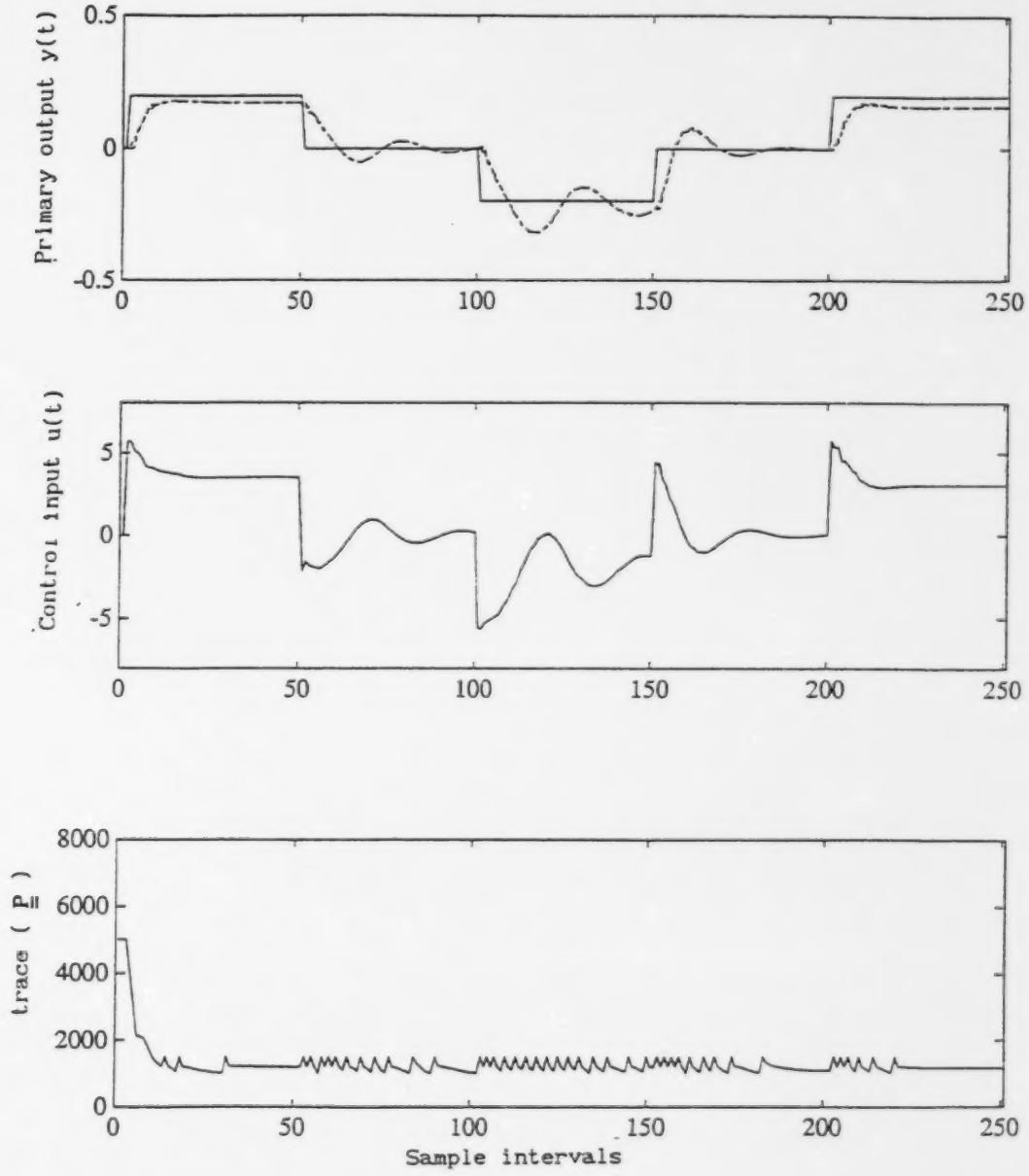

Figure 5.19 Case 2.2b : Servo tracking properties of adaptive multirate inferential controller implemented on a nonlinear process

5.6.2.3 Disturbance rejection

In the presence of time varying load disturbances, it was seen in section 5.5.1.1 that introduction of a residual filter could be effectively used to estimate the effects of the time varying disturbances and prevent biasing of the model parameters. Consequently, if the process outputs are estimated accurately, the feedback control mechanism in the multirate inferential generalized minimum variance controller would minimize the difference between the set point and the estimated output and reject the disturbances.

Case 2.3a Linear model

The simulation results for the case when the process is subjected to a time varying load disturbance using transfer functions relating the process outputs to the disturbance input as given by equations 5.24 and 5.25 are shown in Figure 5.20. It can be seen from Figure 5.20 that the adaptive multirate generalized minimum variance controller rejects the time varying disturbance completely.

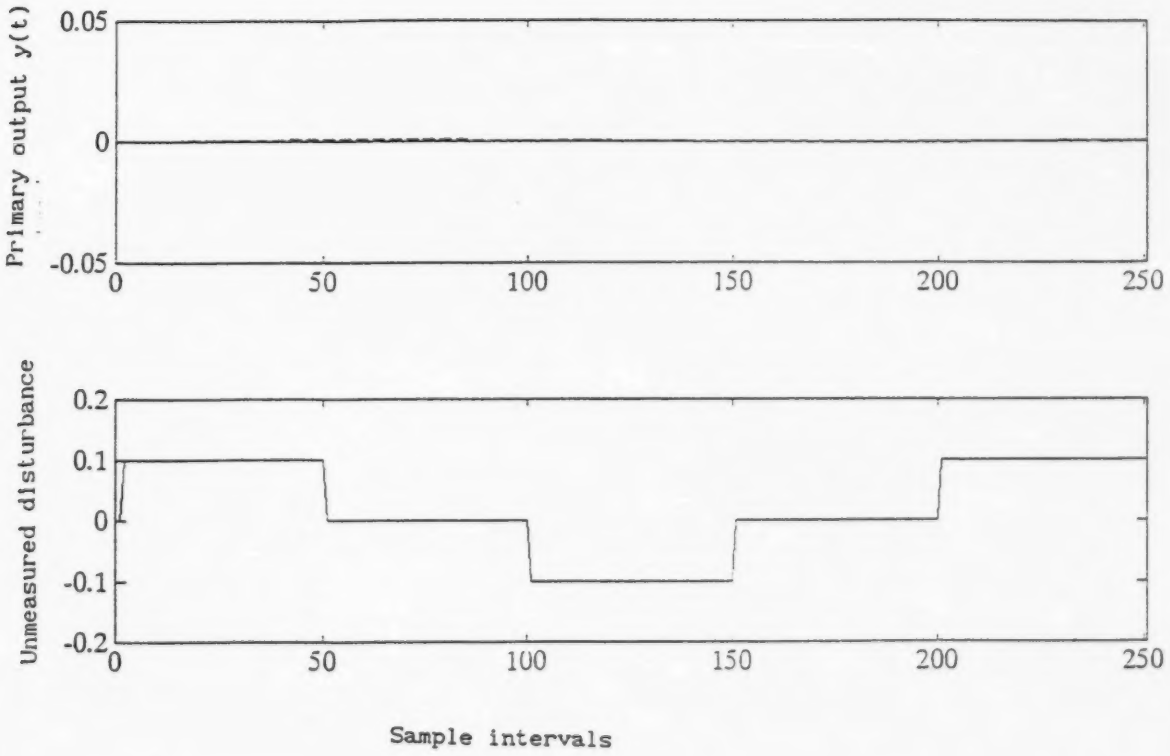

Figure 5.20 Case 2.3a : Disturbance rejection properties of adaptive multirate inferential controller implemented on a linear process

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE STUDY

6.1 Conclusions

The problem of controlling chemical processes with intermittent measurements of the controlled output has been considered in this thesis. The main contribution of the thesis is the development of a multirate Kalman filter to optimally infer intersample values of the controlled output using frequent measurements of related secondary outputs and infrequent measurements of the controlled or primary output.

Inferential estimation algorithms that estimate the intersample values of the primary output by inferring the effects of unmeasured disturbances on the output using secondary measurements (e.g. Brosilow and coworkers, 1978 a,b,c ; Brosilow, 1979; Morari and Stephanopoulus, 1979; Shen and Lee, 1988,1989) suffer from the following difficulties. The major problem is that unmeasured disturbances affecting any process cannot be identified in reality and even if identified, their effect on the process outputs may not be modelled. The design of an inferential estimator, under these circumstances is therefore highly approximate which would lead to offsets in estimation resulting in poor control.

The Kalman filter on the other hand provides a more direct means of estimation. Using the Kalman filter to estimate the

primary output from only secondary measurements is possible (e.g., Guilandoust et al., 1987), if it is assumed that the dynamics of the primary output are completely observable from the secondary measurements. However, it would be very difficult to find a set of such secondary measurements.

The problem of utilizing intermittent or infrequent measurements of the primary output to correct the estimates obtained by inference from the secondary measurements has been solved in the context of multirate flight control applications (Glasson, 1980). The optimal multirate Kalman filter was adapted with some modifications for the purpose of inferential estimation.

Another contribution of this thesis is the development of a minimum variance inferential estimator from the multirate Kalman filter. The minimum variance inferential estimator can be used as a stand-alone estimator (implemented in conjunction with an independent controller) or in an implicit self-tuning control framework, where adaptation of the estimator model parameters can be directly used to calculate the control action. The development of a multirate generalized minimum variance controller used in this study was based on the formulations developed by Clarke and Gawthrop (1975, 1979).

Finally, the relative success or failure of any control strategy can only be measured by its performance. The performance of the adaptive multirate inferential control strategy was tested on linear and nonlinear models of a continuous stirred tank reactor. Simulation results show that the control strategy is

effective while tracking set point changes or regulating the effect of a load disturbance in the presence of noise, process nonlinearities and time varying process parameters. Simulation results showed that the performance of the adaptive inferential controller was superior to the fixed parameter inferential controller, especially when utilizing a nonlinear process model. The performance of the adaptive inferential controller is very much dependent on the performance of the identification algorithm. The recursive least squares algorithm was used to identify the model parameters. From the different modifications that were used to retain the alertness of the recursive least squares algorithm, it was found that the covariance resetting algorithm performed extremely well under the given circumstances.

6.2 Recommendations for Future study

1) Although, it was noted in Chapter 4, that the filter polynomials in the multirate model are directly related to the steady state Kalman gains that can be calculated from the multirate Kalman filter by specifying the noise levels or variances of the process or measurement noise, ad-hoc filtering was employed for the simulation results presented in Chapter 5. This was done because one of the main objectives of this thesis was to demonstrate the effectiveness of the multirate inferential estimator. However, it would be useful, from a theoretical

standpoint, to implement the multirate inferential estimator using filter polynomials calculated from the Kalman gains using the variances of the process and measurement noise as parameters, tuned to obtain the best possible filtering effect.

2) The multirate Kalman filter was developed for the simplest case involving one primary and secondary output. It should, if possible be extended to include more than one of either outputs.

3) It was mentioned earlier that the multirate inferential minimum variance estimator can be used as a stand - alone estimator. This means that any control algorithm can be implemented assuming the inferential estimator provides filtered estimates of the primary output at the frequent sampling rate. This allows us to experiment with different control algorithms. It would be of great practical interest to implement a predictive controller, such as the generalized predictive controller (GPC) with the inferential estimator.

4) A number of process variables in the chemical process industry cannot be easily measured (e.g.chemical composition, biological activity, polymer properties etc.). The inferential estimator can be effectively used as a software sensor to estimate these variables to provide better control. Hence, applications of the inferential estimation and control algorithms developed here should be investigated for the control of such systems.

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APPENDIX A

A sample exercise to illustrate the effect of process and measurement noise characteristics on the solution for the multirate Kalman filter equations

Multirate Kalman filter

System model matrices :

$$\underline{\underline{A}} = \begin{bmatrix} .706 & .0523 \\ -.6217 & 1.0049 \end{bmatrix}$$

$$\underline{\underline{H}}(t) = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array}\right] \quad ; \; t = JT$$

$$\underline{\underline{H}}(t) = [1 \quad 0] \quad ; \; t \neq JT$$

Measurement noise covariance matrix :

$$\underline{\underline{R}}_z(t) = \left[\begin{array}{c|c} 1000 & 0 \\ \hline 0 & 1000 \end{array}\right] \quad ; \quad t = JT$$

$$\underline{\underline{R}}_z(i) = 1000 \quad ; \quad t \neq JT$$

Process noise covariance matrix :

$$\underline{\underline{R}}_w = \left[\begin{array}{c|c} .005 & 0 \\ \hline 0 & .005 \end{array}\right]$$

The multirate Kalman filter given by the equations

$$\underline{\underline{P}}(t/t-1) = \underline{\underline{A}}\,\underline{\underline{P}}(t-1/t-1)\,\underline{\underline{A}} + \underline{\underline{R}}_w$$

$$\underline{K}(t) = \underline{\underline{P}}(t/t-1)\underline{\underline{H}}^T(t)\,[\,\underline{\underline{H}}(t)\,\underline{\underline{P}}(t/t-1)\,\underline{\underline{H}}^T(t) + \underline{\underline{R}}_z(t)\,]^{-1}$$

$$\underline{\underline{P}}(t/t) = \underline{\underline{P}}(t/t-1) - \underline{K}(t)\,\underline{\underline{H}}^T(t)\,\underline{\underline{P}}(t/t-1)$$

is solved for the Kalman gains for J=10 and for different ratios of $\underline{\underline{R}}_w/\underline{\underline{R}}_z$. It can be seen from the results plotted in Figure A.1 that the resulting steady state is periodic. It is also observed that the periodic variation in the Kalman gain K_{22} becomes small as the noise ratio decreases. It can be concluded that an assumption of a constant value for K_{22} would be reasonable for very small values of the process to measurement noise ratio.

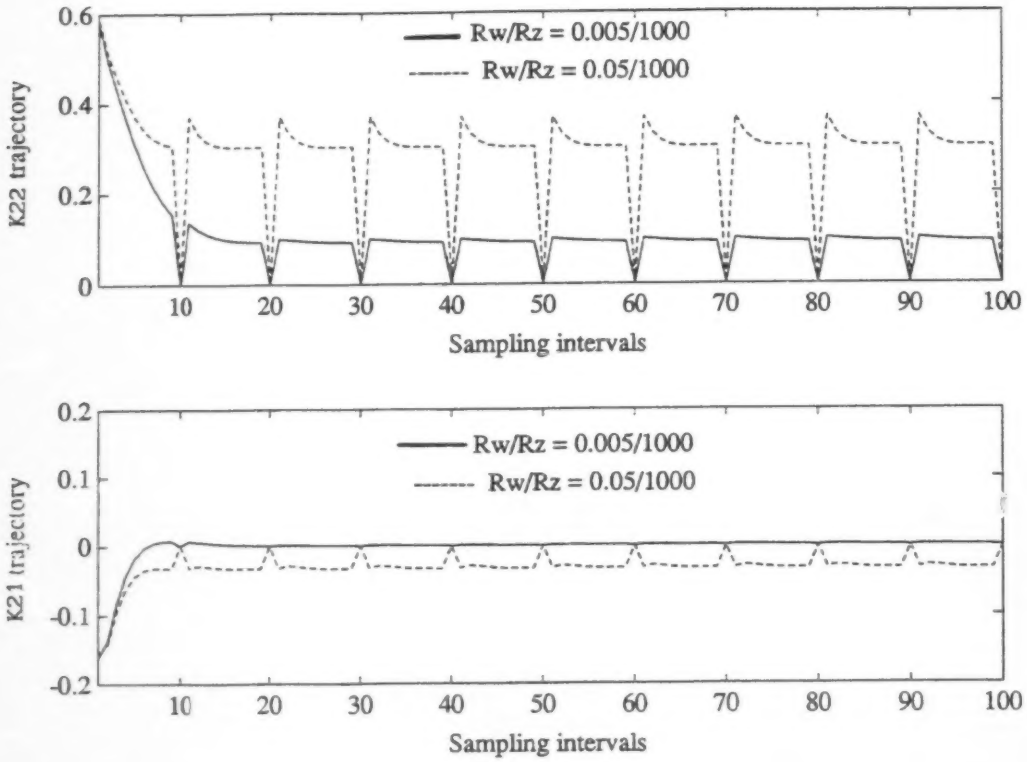

Figure A.1 Kalman gain trajectories for a multirate Kalman filter

APPENDIX B

A representative listing of MATLAB programs related to the implementation of the multirate inferential estimation and control algorithms to the continuous stirred tank reactor (CSTR)

Case 1.1a and 1.1b

```
%                          L_FP_INF_EST.M
%                          --------------


%                                Welcome!
%   This program implements the fixed para. multirate inferential estimation
%   scheme discussed in Chapter 4 (M.Sc. thesis, Iyer, 1992) for the linear
%   process.Files required to run this program are 'reapr1.m','reapr2.m'
%   (Linear process simulators for the primary and secondary processes)
%   Also required are 'miso.m" (RLS multi-input single-output parameter
%   estimator) and 'fildis.m' (filter for the secondary estimator). This
%   program is specifically written for the CSTR implementation and would
%   have to be modified for a different application

% Initialize variables

j=5;                        % Multiple of basic sampling period
u=zeros(1,j+1);             % Control input
y=zeros(1,j+1);             % Primary output measurement
v=zeros(1,j+1);             % Secondary output measurement
vest=zeros(1,j+1);          % Estimate of secondary output
vunc=zeros(1,j+1);          % Secondary meas. minus measurement noise
yest=zeros(1,j+1);          % Primary output estimate
uf=zeros(1,j+1);            % Control input filtered by the secondary observer
                            % polynomial
vf=zeros(1,j+1);            % Similarly filtered secondary output measurement
yf=zeros(1,j+1);            % Similarly filtered primary output measurement
yesf=zeros(1,j+1);          % Similarly filtered primary output estimate
theta=[.1754 .0523 .0369 .0261 .0184 .0130]';  % Parameters for primary
                                               % estimator model
thetav=[0.2049 -.6217 .0501]';                 % Parameters for secondary
                                               % estimator model

nitr=0;                     % No of iterations
```

```
% Store random vectors for process and measurement noise

e1=0.001*rand(251,1);     % Process noise for primary process
e2=.001*rand(251,1);      % Process noise for secondary process
e3=.05*rand(251,1);       % Measurement noise for secondary measurement

% Begin iteration

for i=1:251
nitr=nitr + 1;

% Update input vectors

for k=(j+1):-1:2
u(k)=u(k-1);
uf(k)=uf(k-1);
end

% Specify the most current value for control input

if i<51
u(1)=1;
elseif i<101
u(1)=0;
elseif i<151
u(1)=-1;
elseif i<201
u(1)=0;
else
u(1)=1;
end

uf(1)=fildis(u(1),uf(2));

% Update output vectors
```

```
for k=(j+1):-1:2
y(k)=y(k-1);
v(k)=v(k-1);
vest(k)=vest(k-1);
vunc(k)=vunc(k-1);
yest(k)=yest(k-1);
vf(k)=vf(k-1);
yf(k)=yf(k-1);
yesf(k)=yesf(k-1);
end

% Use the linear process simulators to calculate the current process outputs

[y(1)] = reapr1(y(2),vunc(2),u(1),e1(i,1));
[vunc(1)] = reapr2(vunc(2),y(2),u(1),e2(i,1));

% Calculate the secondary estimate

phiv=[vf(2) yesf(2) uf(1)];
vest(1)=thetav'*phiv';

% In case no filter is used the secondary estimator
% would become:

% vest(1)=[vf(2) yesf(2) uf(1)]*[1.0049 -.6217 .0501]';

v(1)=vunc(1) + e3(i,1) ;
vf(1)=fildis(v(1),vf(2));

% Calculate the primary estimate

phi=[yest(6) vest(2) vest(3) vest(4) vest(5) vest(6)];
yest(1)=theta'*phi';
yesf(1)=fildis(yest(1),yesf(2));
```

```
% Store values for purpose of plotting

yrec(i)=yest(1);
yrec1(i)=y(1);
urec(i)=u(1);
vrec(i)=v(1);
vrec1(i)=vunc(1);
vrec2(i)=vest(1);

end
```

Case 1.2b

```
%                          NL_AD_INF_EST.M
%                          ---------------


%                              Welcome!
%     This program implements the adaptive multirate inferential estimation
%     scheme discussed in Chapter 4 (M.Sc. thesis, Iyer, 1992) for the non
%     linear process simulation of the CSTR. Files required are 'refun.m'
%     (Function specifying the nonlinear mechanistic model of the CSTR
%     expressed as F(x) = 0). Also required are 'miso.m' (RLS identification
%     routine) and 'fildis.m' (filter for secondary estimator).

% Initialize variables

j=5;                          % Multiple of basic sampling period
u=zeros(1,j+1);               % Control input
y=zeros(1,j+1);               % Primary output measurement
v=zeros(1,j+1);               % Secondary output measurement
vest=zeros(1,j+1);            % Estimate of secondary output
vunc=zeros(1,j+1);            % Secondary measurement minus measurement noise
yest=zeros(1,j+1);            % Estimate of secondary output
uf=zeros(1,j+1);              % Control input filtered by secondary observer
                              % polynomial
vf=zeros(1,j+1)               % Similarly filtered sec. output measurement
yf=zeros(1,j+1);              % Similarly filtered primary output measurement
yesf=zeros(1,j+1);            % Similarly filtered primary output estimate
xunc=[0.7127 .9747];          % Operating values of the actual
                              % states or outputs. Note that all variables
                              % defined so far are deviations from the steady
                              % state. Note that the actual states are
                              % initialised to the operating steady state
p=10^3*eye(5);               % Covariance update matrix for primary model ID
pv=10^2* eye(3);             % Covariance update matrix for sec. model ID
```

```
% Variable 'ufor' is operating value for the actual input

global ufor e1fo e2fo

theta(1:5,1)=zeros(5,1);        % Primary estimator model parameters
thetav(1:3,1)=zeros(3,1);       % Secondary estimator model parameters

nitr=0;                         % No. of iterations

% Store random vectors for process and measurement noise

e1=0.001*rand(251,1);           % Process noise for primary process
e2=.001*rand(251,1);            % Process noise for secondary process
e3=.05*rand(251,1);             % Measurement noise for secondary measurement

% Begin iterations

for i=1:251
nitr=nitr + 1;

% Update input vectors
for k=(j+1):-1:2
u(k)=u(k-1);
uf(k)=uf(k-1);
end

% Covariance resetting
if trace(p) < 1000
p=300*eye(5);
pv=100*eye(3);
end

% Specify the current value for the control input (deviation from the SS)

if i<51
```

```
u(1)=1;
elseif i<101
u(1)=0;
elseif i<151
u(1)=-1;
elseif i<201
u(1)=0;
else
u(1)=1;
end

% Calculate the actual value of control input

ufor= -.6395 + u(1);
e1fo=0;
e2fo=0;
uf(1)=fildis(u(1),uf(2));

% Update output vectors

for k=(j+1):-1:2
y(k)=y(k-1);
v(k)=v(k-1);
vest(k)=vest(k-1);
vunc(k)=vunc(k-1);
yest(k)=yest(k-1);
yf(k)=yf(k-1);
vf(k)=vf(k-1);
yesf(k)=yesf(k-1);
end

% Nonlinear process simulation using Runge-Kutta-Fehlberg forward stepping
% method. 'xunc' (1x2) is the initial condition vector which changes after
% each iteration. For the RKF routine, the intial time is always specified
% as 0 and the final time as 0.1. Note that the sampling period used for
% discretization of the continuous time model in Chapter 5 (Iyer, 1992) was
```

```
% 0.1 time units. 'xunr' (nx2) is the output profile vector. 'tt' (nx2) is
% the vector of time steps which depends on the RKF routine which uses
% a variable step size

[tt,xunr]= ode45('refun',0,0.1,xunc);

ste=size(tt);           % 'ste' (1x2) is the no. of time steps assoc. with
                        % xunc(1) and xunc(2)

xunc(1)=xunr(ste(1),1);
xunc(2)=xunr(ste(2),2);

% Calculate the deviations that represent the outputs

y(1)=xunc(1)-.7127;
vunc(1)=xunc(2)-.9747;

% Calculate the secondary estimate

vest(1)=[vf(2) yf(2) uf(1)]*thetav;
v(1)=vunc(1);

x=[y(j+1) vest(2) vest(3) vest(5) vest(6)]'; % Regressor for primary estimator
                                             % model
xv=[vf(2) yesf(2) uf(1)]';                   % Regressor for sec. estimator
                                             % model

yf(1)=fildis(y(1),yf(2));
vf(1)=fildis(v(1),vf(2));

% Calculate primary estimate

phi=[yest(j+1) vest(2) vest(3) vest(5) vest(6)];
yest(1)= (phi*theta) + (.0261*vest(4));      % Note that one parameter assoc.
                                             % with v is fixed for unique
                                             % parameter convergence of
                                             % multirate model
```

```
yesf(1)=fildis(yest(1),yesf(2));

% Store values for purpose of plotting

yrec(i)=yest(1);
yrec1(i)=y(1);
urec(i)=u(1);
vrec(i)=v(1);
vrec1(i)=vunc(1);
vrec2(i)=vest(1);
there1(i)=theta(1,1);
there2(i)=theta(2,1);
there3(i)=theta(3,1);
there4(i)=theta(4,1);
there5(i)=theta(5,1);
therev1(i)=thetav(1,1);
therev2(i)=thetav(2,1);
therev3(i)=thetav(3,1);
ptr(i)=trace(p);
pvtr(i)=trace(pv);

phi1=y(1)-(.0261*vest(4));        % Output for primary estimator model ID
phiv=v(1);                        % Output for secondary estimator model ID

% Estimator model identification

%if nitr==j
[l,p,theta]=miso(phi1,x,theta,p,5);
[lv,pv,thetav]=miso(phiv,xv,thetav,pv,3);
nitr=0;
%end

end
```

Case 2.2a

```
%
%                          LC_AD_INF_CON.M
%                          ---------------
%
%                             Welcome!
%       This program implements the multirate adaptive inferential control
%       strategy discussed in Chapter 4 (M.Sc. thesis, Iyer, 1992) for the
%       linear process simulation of the CSTR. The control scheme is based
%       on the Generalized Minimum Variance Control strategy proposed by
%       Clarke and Gawthrop (1979). The set point scaling routine discussed
%       in Chapter 5 has been incorporated in the program. Files required
%       to run the program are 'reapr1.m', 'reapr2.m' (linear process
%       simulators). Also required are 'miso.m' (RLS identification routine)
%       'fildis.m'(filter for secondary estimator) and 'fildis1.m'(filtering
%       due to the denominator polynomial of the auxilliary transfer function
%       in the auxilliary output prediction.


% Initialize variables

j=5;                          % Multiple of basic sampling period
pn=0.8246;                    % Parameter assoc. with numerator polynomial
                              % of auxilliary output transfer function
pd=0.1754;                    % Parameter assoc. with denominator polynomial
                              % of auxilliary output transfer function
u=zeros(1,j+1);               % Control input
y=zeros(1,(2*j)+1);           % Primary output measurement
ysp=zeros(1,j+1);             % Set point
yspsc=zeros(1,j+1);           % Scaled set point
yspf=zeros(1,j+1);            % Set point filtered by secondary observer
                              % polynomial
psi = zeros(1,j+1);           % Auxilliary output
psiest=zeros(1,j+1);          % Estimate of auxilliary output
v=zeros(1,j+1);               % Secondary output measurement
```

```
vest=zeros(1,j+1);              % Estimate of secondary output
vunc=zeros(1,j+1);              % Secondary measurement minus measurement noise
yest=zeros(1,(2*j)+1);          % Estimate of primary output
yff =zeros(1,(2*j)+1);          % Primary output measurement filtered by
                                % denominator polynoimial of auxilliary
                                % transfer function
yestff=zeros(1,(2*j)+1);        % Primary estimate filtered by denominator
                                % polynomial of auxilliary transfer function
uf=zeros(1,j+1);                % Control input filtered by secondary observer
                                % polynomial
vpr=zeros(1,j+1);               % Prediction of secondary output
vf=zeros(1,j+1);                % Secondary measurement filtered by secondary
                                % observer polynomial
yf=zeros(1,(2*j)+1);            % Similarly filtered primary output measurement
yesf=zeros(1,(2*j)+1);          % Similarly filtered primary output estimate

p=10^3*(eye(5));                % Covariance update matrix for primary model
pv=10^3*(eye(3));               % Covariance update matrix for secondary model

lamda=0.05;                     % Control input weighting
sca= 1 + 8.466*(lamda);         % Scaling factor

theta1(1:5,1) = zeros(5,1);     % First 5 parameters of the primary estimator
                                % or predictor model
thetapr(1:5,1) = theta1;
thetapr(6:7,1) = [0.0184*pn .0130*pn]';  % Parameters that are fixed during ID
thetav((1:3),1) = zeros(3,1);            % Parameters of secondary estimator
                                         % or predictor model

nitr=0;                         % No. of iterations

% Store random vectors for process and measurement noise

e1=.0001*rand(251,1);
e2=.00001*rand(251,1);
e3=.001*rand(251,1);
```

```
% Begin iteration

for i=1:251
nitr=nitr + 1;

% Update input vector, setpoint and prediction

for k=(j+1):-1:2
u(k)=u(k-1);
uf(k)=uf(k-1);
vpr(k)=vpr(k-1);
ysp(k)=ysp(k-1);
yspsc(k)=yspsc(k-1);
yspf(k)=yspf(k-1);
end

% Specify the current setpoint

if i<51
if i<2
ysp(1)=0.0;
else
ysp(1)=0.02;
end
elseif i<101
ysp(1)=0.0;
elseif i<151
ysp(1)=-0.02;
elseif i<201
ysp(1)=0.0;
elseif i<251
ysp(1)=0.02;
end
yspsc(1) = sca*ysp(1);
yspf(1)=fildis(yspsc(1),yspf(2));
```

```
% Calculate the control input

thetapr( (1:5),1) = theta1;
xtilde=[yestff(5) yestff(10) vpr(2) vpr(3) vpr(4) vpr(5) vpr(6)]';
u(1)=(yspsc(1)-(thetapr'*xtilde))/lamda;
uf(1)=fildis(u(1),uf(2));

% Calculate secondary prediction

vpr(1)=thetav'*[ vf(2) yspf(2) uf(1)]';

% Update the output vectors

for k=(j+1):-1:2
v(k)=v(k-1);
vest(k)=vest(k-1);
vunc(k)=vunc(k-1);
psiest(k)=psiest(k-1);
vf(k)=vf(k-1);
psi(k)=psi(k-1);
end
for k=((2*j) + 1):-1:2
y(k)=y(k-1);
yest(k)=yest(k-1);
yesf(k)=yesf(k-1);
yestff(k)=yestff(k-1);
yf(k)=yf(k-1);
yff(k)=yff(k-1);
end

% Calculate the outputs from linear process simulators

[y(1)] = reapr1(y(2),vunc(2),u(1),e1(i,1) );
[vunc(1)] = reapr2(vunc(2),y(2),u(1),e2(i,1));
```

```
% Estimate the secondary output

phiv=[vf(2) yesf(2) uf(1)];
vest(1)=thetav'*phiv';
v(1)=vunc(1) + e3(i,1) ;
vf(1)=fildis(v(1),vf(2));

% Estimate the auxilliary output

phi=[yestff(6) yestff(11) vest(2) vest(3) vest(4) vest(5) vest(6)]';
psiest(1)=thetapr'*phi;
psi(1) = (pn*y(1)) + (pd*y(j+1));
yff(1) = fildis1(y(1),yff(6));

% Estimate the primary output

yest(1)=(psiest(1) - (pd*psiest(j+1)))/pn;
yesf(1)=fildis(yest(1),yesf(2));
yestff(1)=fildis1(yest(1),yestff(6));

x2 = [vf(2) yesf(2) uf(1)]';      % Regressor for secondary model ID
phi2 = v(1);                      % Output for secondary model ID

x1=[yff(6) yff(11) vest(2) vest(3) vest(4)]'; % Regressor for primary model ID
phi1 = psi(1) - ([thetapr(6,1) thetapr(7,1)]*[vest(5) vest(6)]'); % Output for
                                                                  % primary
                                                                  % model ID
% Store values for purpose of plotting

yrec(i)=yest(1);
psirec(i)=psiest(1);
yrec1(i)=y(1);
urec(i)=u(1);
vrec(i)=v(1);
vrec1(i)=vunc(1);
vrec2(i)=vest(1);
```

```
there1(i)=theta1(1);
there2(i)=theta1(2);
there3(i)=theta1(3);
there4(i)=theta1(4);
there5(i)=theta1(5);
ysprec(i)= ysp(1);
ptr(i)=trace(p);

% Estimator or predictor model identification

%if nitr==j
[l,p,theta1]=miso(phi1,x1,theta1,p,5);
[lv,pv,thetav]=miso(phi2,x2,thetav,pv,3);
%nitr=0;
%end

end
```

Auxilliary programs

```
%             REAPR1.M
%             ---------
function[y1]=reapr1(y2,v2,u1,di)
ay=[1 -.7060];
by=[0];
my=[.0523];
phiy=[-ay(2) my(1) by(1) 1];
thety=[y2 v2 u1 di];
y1=phiy*thety';

%             REAPR2.M
%             --------
function[vunc1]=reapr2(vunc2,y2,u1,dii)
av=[1 -1.0049];
bv=[.0501];
mv=[-.6217];
thetv=[-av(2) mv(1) bv(1) 1 ];
phiv=[vunc2 y2 u1 dii];
vunc1=phiv*thetv';

%             REFUN.M
%             --------
function xdot = refun(t,x);
da = 1;
h=2.5;
gam=13.4;
bet=0.5;
xdot(1)=-x(1) + (da*(1-x(1))*(exp((x(2)/(1+(x(2)/gam))))))+ e1fo;
xdot(2)=-(x(2)*(1+bet))+(h*da*(1-x(1))*(exp((x(2)/(1+(x(2)/gam))))))
        +(bet*ufor)
xdot(2)=xdot(2) + e2fo;
```

```
%                    MISO.M
%                    ------

function [l,pf,thetf]=miso(yf,phif,thetf,pf,n)

% Calculate the values of variable forgetting factor

%lemdanum=(yf-thetf'*phif)^2;
%lemdaden=(1.0+phif'*pf*phif)*sigma;
%lemda=1.0-lemdanum/lemdaden;
lemda=1;

% Calculate the gain vector

l=(pf*phif)/(lemda+phif'*pf*phif);
w=pf-(l*phif'*pf);
if w< 99*eye(n)
pf=w/lemda;
else
pf=w;
end
% Calculate the value of covariance update
%pnum=pf*phif*phif'*pf;
%pden=lemda+phif'*pf*phif;
%pf=(pf-pnum/pden)/lemda;

% Updating the parameters

thetf=thetf+l*(yf-thetf'*phif);

clc
end
```